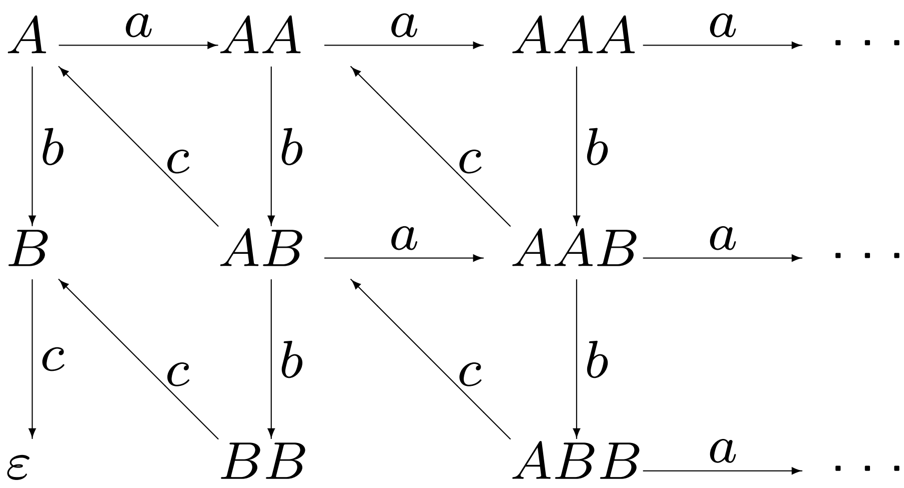
Basic Parallel Process

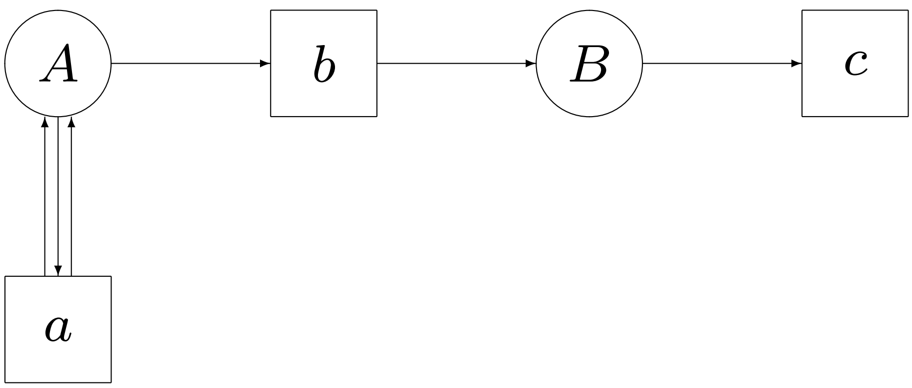
$A \xrightarrow{a} AA$

$A \xrightarrow{b} B$

$B \xrightarrow{c} \varepsilon$



Petri Net



Norm

- norm $\alpha = \min\{|w| \mid \alpha \stackrel{w}{\rightarrow} \varepsilon\}$
- $\text{norm}_Q\ \alpha = \min\{|w| \mid \alpha \stackrel{w}{\rightarrow} \beta \wedge \beta \cap Q = \emptyset\}$

Linear function

$f : \Pi^* \rightarrow \mathbf{N}_\omega$ such that for each X there exists c_X such that

$$f(\alpha) = \sum_X c_X \cdot |\alpha|_X$$

Each norm_Q is a linear function.

What is a bisimilarity?

A maximal relation $\sim$ between processes where if $r_1 \sim r_1'$, then:
$\forall a, r_2 : r_1 \xrightarrow{a} r_2 \Rightarrow (\exists r_2' : r_1' \xrightarrow{a} r_2' \wedge r_2 \sim r_2')$,
$\forall a, r_2' : r_1' \xrightarrow{a} r_2' \Rightarrow (\exists r_2 : r_1 \xrightarrow{a} r_2 \wedge r_2 \sim r_2')$.

Question:

Given two processes α, β of a BPP, are they bisimilar?

General bisimilarity on BPP: **PSPACE-complete**
Bisimilarity on *normed* BPP: **P-complete**

Hirshfeld-Jerrum-Moller algorithm

A *decomposition* $\mathcal{D}$:
for each elementary process X:

- either $\mathcal{D}(X) = X$ (a *prime*),
- or $\mathcal{D}(X) = P_1^{x_1} \dots P_n^{x_n}$ where $P_1, \dots, P_n$ are primes and $X \sim P_1^{x_1} \dots P_n^{x_n}$.

$\mathcal{D}(X_1 \dots X_n) = \mathcal{D}(X_1) \dots \mathcal{D}(X_n)$.

Then:

- $\mathcal{D}(\alpha) \sim \alpha$,
- $\alpha \sim \beta$ iff $\mathcal{D}(\alpha) = \mathcal{D}(\beta)$.

$A \to a \mid aB$
$B \to b$
$X \to aB \mid bA$
$Y \to aAB \mid bAA \mid aX \mid aBX$

	A	B	X	Y
norm	1	1	2	3

We see:

- A, B are primes
- $\mathcal{D}(X) = AB$, $\mathcal{D}(Y) = AAB$?
- the rule $A \to aB$ causes $\mathcal{D}(X) = AB$ to fail because there is no $X \to a\alpha$ such that $\mathcal{D}(\alpha) = BB$ (to be able to perform $X \sim AB \to aBB$)

$$\begin{array}{lllllll} AB & \xrightarrow{a} & BB & \xrightarrow{b} & B & \xrightarrow{b} & \varepsilon \\ X & \xrightarrow{a} & B & \xrightarrow{b} & \varepsilon & \not\xrightarrow{b} & \end{array}$$

finish: A, B, X are primes, $\mathcal{D}(Y) = AX$

Jančar algorithm

$A \to a \mid aB$
$B \to b$
$X \to aB \mid bA$
$Y \to aAB \mid bAA \mid aX \mid aBX$

T_1	T_2
$A \xrightarrow{a} \varepsilon$	$B \xrightarrow{b} \varepsilon$
$A \xrightarrow{a} B$	$X \xrightarrow{b} A$
$X \xrightarrow{a} B$	$Y \xrightarrow{b} AA$
$Y \xrightarrow{a} AB$	
$Y \xrightarrow{a} X$	
$Y \xrightarrow{a} BX$	

$\text{norm}_{\{B,X,Y\}}$: $c_A = 0,\ c_B = 1,\ c_X = 1,\ c_Y = 1$

$\text{norm}_{\{A,X,Y\}}$: $c_A = 1,\ c_B = 0,\ c_X = 1,\ c_Y = 2$

$$\delta_L(X \to \alpha) = \sum_{Y \in \alpha} c_Y \cdot |\alpha|_Y - c_X$$

$\delta_{L_1}(A \xrightarrow{a} \varepsilon) = 0$	$\delta_{L_2}(A \xrightarrow{a} \varepsilon) = -1$
$\delta_{L_1}(A \xrightarrow{a} B) = 1$	$\delta_{L_2}(A \xrightarrow{a} B) = -1$
$\delta_{L_1}(X \xrightarrow{a} B) = 0$	$\delta_{L_2}(X \xrightarrow{a} B) = -1$
$\delta_{L_1}(Y \xrightarrow{a} AB) = 0$	$\delta_{L_2}(Y \xrightarrow{a} AB) = -1$
$\delta_{L_1}(Y \xrightarrow{a} X) = 0$	$\delta_{L_2}(Y \xrightarrow{a} X) = -1$
$\delta_{L_1}(Y \xrightarrow{a} BX) = 1$	$\delta_{L_2}(Y \xrightarrow{a} BX) = -1$
$\delta_{L_1}(B \xrightarrow{b} \varepsilon) = -1$	$\delta_{L_2}(B \xrightarrow{b} \varepsilon) = 0$
$\delta_{L_1}(X \xrightarrow{b} A) = -1$	$\delta_{L_2}(X \xrightarrow{b} A) = 0$
$\delta_{L_1}(Y \xrightarrow{b} AA) = -1$	$\delta_{L_2}(Y \xrightarrow{b} AA) = 0$

T_1	T_2	T_3
$A \xrightarrow{a} \varepsilon$	$B \xrightarrow{b} \varepsilon$	$A \xrightarrow{a} B$
$X \xrightarrow{a} B$	$X \xrightarrow{b} A$	$Y \xrightarrow{a} BX$
$Y \xrightarrow{a} AB$	$Y \xrightarrow{b} AA$	
$Y \xrightarrow{a} X$		

...

T_1	T_2	T_3	T_4	T_5
$A \xrightarrow{a} \varepsilon$ $Y \xrightarrow{a} X$	$X \xrightarrow{a} B$ $Y \xrightarrow{a} AB$	$A \xrightarrow{a} B$ $Y \xrightarrow{a} BX$	$X \xrightarrow{b} A$ $Y \xrightarrow{b} AA$	$B \xrightarrow{b} \varepsilon$

$$T_1 = \mathsf{norm}_{\{A,Y\}} : c_A = 1,\ c_B = 0,\ c_X = 0,\ c_Y = 1$$

$$T_2 = \mathsf{norm}_{\{X,Y\}} : c_A = 0,\ c_B = 0,\ c_X = 1,\ c_Y = 1$$

$$T_3 = \mathsf{norm}_{\{B\}} : c_A = 0,\ c_B = 1,\ c_X = 0,\ c_Y = 0$$

$$\alpha \sim \beta$$

$$\Updownarrow$$

$$\forall i : \sum_X c_X(T_i) \cdot |\alpha|_X = \sum_X c_X(T_i) \cdot |\beta|_X$$