

Tutorial 6 – First Order Predicate Logic

Exercise 1: Express the following formulas verbally, assuming that the predicate P means to like (who, whom), the individual constant m means Mary and the individual constant k Charles.

- a) $\exists x \exists y P(x, y)$
- b) $\exists x \forall y P(x, y)$
- c) $\exists y \forall x P(x, y)$
- d) $\forall x \exists y P(x, y)$
- e) $\forall x \forall y P(x, y)$
- f) $\forall x P(x, m)$
- g) $\forall y P(k, y)$

Exercise 2: Formalize following sentences into the language of First order predicate logic. Negate formally and verbally.

- a) Everyone is in Mexico.
- b) Someone is water goblin.
- c) Each water goblin is wet.
- d) All water goblins are not wet.
- e) Not all water goblins are wet.
- f) No water goblin is wet.
- g) At least someone loves his mother.
- h) Some people do not like logic.
- i) Some people preach water but drink wine.
- j) No educated man fell from heaven.
- k) All man a football players and beer drinkers.
 - l) Some beer drinkers are not football players.
- m) Natural numbers are greater than or equal to 0.
- n) All even numbers are divisible by 2.
- o) Every natural number is an integer.

- p) There is an even prime number.
- q) Prime numbers greater than 2 are odd.
- r) No positive number is negative.
- s) Someone is thinking that he is smarter than everyone else.
- t) Two roosters in one dump hate each other.
- u) When the lords quarrel, the peasants loose their hair.
- v) All students like their ILT tutor.

Exercise 3: For the following sequences of symbols, decide whether it is a well-formed formula of FOL (use common parentheses conventions). In case of well-formed formula, do the following:

- Decide, which symbols in the formulas are predicate symbols and which are terms. To each of them, determine the number of their arguments (arity).
- For each occurrence of a variable, identify whether it is free or bound occurrence.

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| a) P | k) $f(x, y)$ |
| b) $P(P(x, y), z)$ | l) $\neg P(x) \wedge \exists x Q(x)$ |
| c) $x(P)$ | m) $\forall x \forall y [P(x) \vee P(x, y)]$ |
| d) Pxy | n) $\forall x \exists y P(x)$ |
| e) $\forall P(x)$ | o) $\forall x \exists x P(x)$ |
| f) $P(x) \wedge \exists$ | p) $\forall x \exists y P(R(x, y))$ |
| g) $\exists x R(x, y)$ | q) $\forall x \exists y P(R(\neg x, y))$ |
| h) $\forall z [R(x, y) \supset R(y, x)]$ | r) $\exists x P(u, z) \wedge \forall y [\neg Q(y, x) \vee P(y, z)]$ |
| i) $\neg \exists x [\neg P(x, y) \supset R(u, x)]$ | s) $\forall x \forall x \forall x \forall x \forall x \forall x \forall x \forall x \forall x S(x, x, x)$ |
| j) $P(f(x, y), a)$ | t) $\exists x [\forall y (\neg P(x, y)) \supset \forall y Q(y)] \equiv R(x, y)$ |

Exercise 4: Decide, which interpretation is a model of which of the following formulas.
Interpretation:

- universum $U = \{a, b, c\}$
- $P^U \subseteq U = \{a, c\}$

- $Q^U = \emptyset$
- $R^U \subseteq U \times U = \{(a, c), (b, b), (b, c), (c, a)\}$
- $L^U \subseteq U \times U = \{(a, a), (b, b), (c, c), (a, b), (b, c), (c, a)\}$
- $f^U : U \times U \rightarrow U = \{(a, b, c), (a, c, c), (b, c, c), (b, a, c), (c, a, c), (c, b, c), (a, a, c), (b, b, c), (c, c, c)\}$

Formulas:

- $\forall x P(x)$
- $\exists x \neg Q(x)$
- $\forall x \exists y R(x, y)$
- $\forall x L(x, x)$
- $\forall x \forall y [R(x, y) \supset R(y, x)]$
- $\forall x \forall y P(f(x, y))$
- $\exists x [Q(x) \wedge \forall y R(y, x)]$
- $\forall x \forall y [R(x, y) \supset L(y, x)]$

Exercise 5: For the following formulas, define their models and interpretations that are not a models.

- $\exists x [A(x) \wedge B(x)]$
- $\forall x [A(x) \supset B(x)]$
- $\exists x [A(x) \wedge \neg B(x)]$
- $\forall x R(x, f(x))$
- $\forall x \forall y [R(x, y) \supset R(y, x)]$