Tutorial 6 – First Order Predicate Logic

Exercise 1: Express the following formulas verbally, assuming that the predicate P means to like (who, whom), the individual constant m means Mary and the individual constant k Charles.

a) $\exists x \exists y P(x, y)$

b) $\exists x \forall y P(x, y)$

c) $\exists y \forall x P(x, y)$

d) $\forall x \exists y P(x, y)$

e) $\forall x \forall y P(x, y)$

f) $\forall x P(x, m)$

g) $\forall y P(k, y)$

Exercise 2: Formalize following sentences into the language of First order predicate logic. Negate formally and verbally.

a) Everyone is in Mexico.

b) Someone is water goblin.

c) Each water goblin is wet.

d) All water goblins are not wet.

e) Not all water goblins are wet.

f) No water goblin is wet.

g) At least someone loves his mother.

h) Some people do not like logic.

i) Some people preach water but drink wine.

j) No educated man fell from heaven.

k) All man a football players and beer drinkers.

l) Some beer drinkers are not football players.

m) Natural numbers are greater than or equal to 0.

n) All even numbers are divisible by 2.

o) Every natural number is an integer.

p) There is an even prime number.

q) Prime numbers greater than 2 are odd.

r) No positive number is negative.

s) Someone is thinking that he is smarter than everyone else.

t) Two roosters in one dump hate each other.

u) When the lords quarrel, the peasants loose their hair.

v) All students like their ILT tutor.

Exercise 3: For the following sequences of symbols, decide whether it is a well-formed formula of FOL (use common parentheses conventions). In case of well-formed formula, do the following:

- Decide, which symbols in the formulas are predicate symbols and which are terms. To each of them, determine the number of their arguments (arity).
- For each occurrence of a variable, identify whether it is free or bound occurrence.

a) P

b) $P(P(x,y),z)$

c) $x(P)$

d) Pxy

e) $\forall P(x)$

f) $P(x) \wedge \exists$

g) $\exists x R(x,y)$

h) $\forall z[R(x,y) \supset R(y,x)]$

i) $\neg\exists x[\neg P(x,y) \supset R(u,x)]$

j) $P(f(x,y),a)$

k) $f(x,y)$

l) $\neg P(x) \wedge \exists x Q(x)$

m) $\forall x \forall y[P(x) \vee P(x,y)]$

n) $\forall x \exists y P(x)$

o) $\forall x \exists x P(x)$

p) $\forall x \exists y P(R(x,y))$

q) $\forall x \exists y P(R(\neg x,y))$

r) $\exists x P(u,z) \wedge \forall y[\neg Q(y,x) \vee P(y,z)]$

s) $\forall x \forall x \forall x \forall x \forall x \forall x \forall x \forall x \forall x S(x,x,x)$

t) $\exists x[\forall y(\neg P(x,y)) \supset \forall y Q(y)) \equiv R(x,y)]$

Exercise 4: Decide, which interpretation is a model of which of the following formulas.
Interpretation:

- universum $U = \{a,b,c\}$
- $P^U \subseteq U = \{a,c\}$

- $Q^U = \emptyset$
- $R^U \subseteq U \times U = \{(a, c), (b, b), (b, c), (c, a)\}$
- $L^U \subseteq U \times U = \{(a, a), (b, b), (c, c), (a, b), (b, c), (c, a)\}$
- $f^U : U \times U \to U$ = {(a,b,c), (a,c,c), (b,c,c), (b,a,c), (c,a,c), (c,b,c), (a, a, c), (b,b, c), (c, c, c)}

Formulas:

a) $\forall x P(x)$

b) $\exists x \neg Q(x)$

c) $\forall x \exists y R(x, y)$

d) $\forall x L(x, x)$

e) $\forall x \forall y [R(x, y) \supset R(y, x)]$

f) $\forall x \forall y P(f(x, y))$

g) $\exists x [Q(x) \wedge \forall y R(y, x)]$

h) $\forall x \forall y [R(x, y) \supset L(y, x)]$

Exercise 5: For the following formulas, define their models and interpretations that are not a models.

a) $\exists x [A(x) \wedge B(x)]$

b) $\forall x [A(x) \supset B(x)]$

c) $\exists x [A(x) \wedge \neg B(x)$

d) $\forall x R(x, f(x))$

e) $\forall x \forall y [R(x, y) \supset R(y, x)]$