

# Resolving Topic-Focus Ambiguities in Natural Language

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## 1. Introduction

Natural language has features that are not found in logically perfect artificial languages. One such feature is *redundancy*, where two or more terms/expressions share exactly the same semantic and logical (but perhaps not pragmatic or rhetoric) properties. Another feature is its converse, namely *ambiguity*, where one term/expression has more than one meaning. A logical analysis of such a piece of natural language will typically translate each of its unambiguous meanings into logically perfect notation. Frege's Begriffsschrift was the first major attempt in modern logic to create such a notation (though he primarily intended it for mathematical language).<sup>1</sup> There are various origins and various manifestations of ambiguity, not least cases bearing on quantifier scopes, like "Every boy dances with one girl". Another sort of example is "John loves his wife, and so does Peter", which is ambiguous between Peter loving John's wife and Peter loving his own wife, because it is ambiguous which property 'so' picks up.<sup>2</sup> A third, and perhaps less-noticed, sort of ambiguity is pivoted on whether the *topic* or the *focus* of a sentence is highlighted. For instance, "John only introduced Bill to Sue", to use Hajičová's example,<sup>3</sup> lends itself to two different kinds of construal: "John did not introduce other people to Sue except for Bill" and "The only person Bill was introduced to by John was Sue". There are two sentences whose semantics, logical properties and consequences only partially overlap. A similar phenomenon also crops up in the case of propositional attitudes and their less-attended 'cousins' of notional attitudes like seeking and finding, calculating and proving.

In this chapter I will deal in particular with ambiguities in natural language exemplifying the difference between *topic* and *focus articulation* within a sentence. This difference is closely related to the disambiguation stemming from supposition *de dicto* and *de re* with which a particular expression is used. I will show that whereas articulating the topic of a sentence activates a presupposition, articulating the focus frequently yields merely entailment. Based on an analysis of topic-focus articulation, I propose a solution to the almost hundred-year old dispute over Strawsonian vs. Russellian definite descriptions.<sup>4</sup> The point of departure is

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<sup>1</sup> See (Frege, 1884).

<sup>2</sup> See (Neale, 2004), and also (Duží & Jespersen, submitted).

<sup>3</sup> See (Hajičová, 2008).

<sup>4</sup> See, for instance, (Donellan, 1966); (Fintel, 2004); (Neale 1990); (Russell, 1905, 1957); (Strawson 1950, 1964).

that sentences of the form “The  $F$  is a  $G$ ” are ambiguous. Their ambiguity is, in my view, not rooted in a shift of meaning of the definite description ‘the  $F$ ’. Rather, the ambiguity stems from different topic-focus articulations of such sentences. Russell and Strawson took themselves to be at loggerheads; whereas, in fact, they spoke at cross purposes. The received view still tends to be that there is room for at most one of the two positions, since they are deemed incompatible. And they are, of course, incompatible – if they must explain the same set of data. But they should not, in my view. One theory is excellent at explaining one set of data, but poor at explaining the data that the other theory is excellent at explaining; and *vice versa*. My novel contribution advances the research into definite descriptions by pointing out how progress has been hampered by a false dilemma and *how to move beyond that dilemma*. The point is this. If ‘the  $F$ ’ is the topic phrase then this description occurs with *de re* supposition and Strawson’s analysis appears to be what is wanted. On this reading the sentence *presupposes* the existence of the descriptum of ‘the  $F$ ’. The other option is ‘ $G$ ’ occurring as topic and ‘the  $F$ ’ as focus. This reading corresponds to Donnellan’s *attributive* use of ‘the  $F$ ’ and the description occurs with *de dicto* supposition. On this reading the Russellian analysis gets the truth-conditions of the sentence right. The existence of a unique  $F$  is merely entailed.

Ancillary to my analysis is a *general analytic schema* of sentences coming with a presupposition. This analysis makes use of a definition of the ‘if-then-else’ connective known from programming languages. A broadly accepted view of the semantic nature of this connective is that it is a so-called non-strict function that does not comply with the principle of compositionality. However, the semantic nature of the connective is contested among computer scientists. I will show — and this is also a novel contribution of mine — that there is no cogent reason for embracing a non-strict definition and context-dependent meaning, *provided* a higher-order logic making it possible to operate on hyperintensions is applied. The framework of Tichý’s Transparent Intensional Logic (TIL) possesses sufficient expressive power, and will figure as background theory throughout my exposition.<sup>5</sup>

Tichý’s TIL was developed simultaneously with Montague’s Intensional Logic, IL.<sup>6</sup> The technical tools of disambiguation will be familiar from IL, with two exceptions. One is that we  $\lambda$ -bind separate variables  $w, w_1, \dots, w_n$  ranging over possible worlds and  $t, t_1, \dots, t_n$  ranging over times. This dual binding is tantamount to *explicit intensionalization and temporalization*. The other exception is that *functional application* is the logic both of extensionalization of intensions (functions from possible worlds) and of predication.<sup>7</sup> Application is symbolized by square brackets, ‘[...]’. Intensions are extensionalized by applying them to worlds and times, as in  $[[Intension\ w]\ t]$ , abbreviated by subscripted terms for world and time variables:  $Intension_{wt}$  is the extension of the generic intension  $Intension$  at  $\langle w, t \rangle$ . Thus, for instance, the extensionalization of a property yields a set (possibly an empty one), and the extensionalization of a proposition yields a truth-value (or no value at all). A general objection to Montague’s IL is that it fails to accommodate *hyperintensionality*, as indeed any formal logic interpreted set-theoretically is bound to unless a domain of primitive hyperintensions is added to the frame. Any theory of natural-language analysis needs a hyperintensional (preferably procedural) semantics in order to crack the hard nuts

<sup>5</sup> For details on TIL see, in particular, (Duží et al., 2010a); (Tichý, 1988, 2004).

<sup>6</sup> For a detailed critical comparison of TIL and Montague’s IL, see (Duží et al., 2010a, § 2.4.3);

<sup>7</sup> For details, see (Jespersen, 2008).

of natural language semantics. In global terms, without procedural semantics TIL is an anti-contextualist (i.e., transparent), explicitly intensional modification of IL. With procedural semantics, TIL rises above the model-theoretic paradigm and joins instead the paradigm of hyperintensional logic and structured meanings.

The structure of this chapter is as follows. In Section 2 I briefly summarise the history of the dispute between Russell and Strawson (as well as their proponents and opponents) on the semantic character of sentences containing definite descriptions. Section 3 is an introduction to TIL. In paragraph 3.1 I introduce the semantic foundations of TIL and in 3.2 its logical foundations. Sections 4 and 5 contain the main results of this study. In Section 4 I propose a solution to the dispute over Strawsonian vs. Russellian definite descriptions. Paragraph 4.1 is an introduction to the problem of ambiguities stemming from different topic-focus articulation and a solution based on this distinction is proposed in paragraph 4.2. Section 5 generalizes the method of topic-focus disambiguation to sentences containing not only definite descriptions but also general terms occurring with different suppositions. To this end I make use of the strict analysis of the *if-then-else* function that is defined in paragraph 5.1. The method is then illustrated by analysing some more examples in paragraph 5.2. Finally, Section 6 summarizes the results.

## 2. Russell vs. Strawson on definite descriptions

There is a substantial difference between proper names and definite descriptions. This distinction is of crucial importance due to their vastly different logical behaviour. Independently of any particular theory of proper names, it should be granted that a *proper* proper name (as opposed to a definite description grammatically masquerading as a proper name) is a rigid designator of a numerically particular individual. On the other hand, a definite *description* like, for instance, ‘the Mayor of Dunedin’, ‘the King of France’, ‘the highest mountain on Earth’, etc., offers an *empirical criterion* that enables us to establish which individual, if any, satisfies the criterion in a particular state of affairs.

The contemporary discussion of the distinction between names and descriptions was triggered by Russell (1905). Russell’s key idea is the proposal that a sentence like

(1) “The *F* is a *G*”, containing a definite description ‘the *F*’ is understood to have, in the final analysis, the logical form

(1’)  $\exists x (Fx \wedge \forall y (Fy \supset x=y) \wedge Gx)$ , rather than the logical form  $G(x Fx)$ .

Though Russell’s quantificational theory remains to this day a strong rival of referential theories, it has received its fair share of *criticism*. *First*, Russell’s translation of simple sentences like “The *F* is a *G*” into the molecular form “There is at least one *F* and at most one thing is an *F* and that thing is a *G*” is rather enigmatic, because Russell disregards the standard constraint that there must be a fair amount of structural similarity between analysandum and analysans. *Second*, Russell proposed the elimination of Peano’s descriptive operator ‘*t*’ understood as ‘the only’, and deprived definite descriptions of their self-contained meaning. *Third*, Russell simply got the truth-conditions wrong in important cases of using descriptions when there is no such thing as the unique *F*. This criticism was launched by Strawson who in (1950) objected that Russell’s theory predicts the wrong truth-conditions for sentences like “The present King of France is bald”. According to Russell’s

analysis, this sentence is false, but according to Strawson, this outcome does not conform to our intuitions about its truth or falsity. In Strawson's view, the sentence can be neither true nor false whenever there is no King of France. Obviously, in such a state of affairs the sentence is not true. However, if it were false then its negation, "The King of France is not bald", would be true, which entails that there is a unique King of France, contrary to the assumption that there is none. Strawson held that sentences like these *not only entail* the existence of the present King of France, but also *presuppose* his existence. If 'the present King of France' fails to refer, then the presupposition is false and the sentence fails to have a determinate truth value.<sup>8</sup>

Russell (1957) in response to Strawson's criticism argued that, despite Strawson's protests, the sentence was in fact false:

Suppose, for example, that in some country there was a law that no person could hold public office if he considered it false that the Ruler of the Universe is wise. I think an avowed atheist who took advantage of Mr. Strawson's doctrine to say that he did not hold this proposition false would be regarded as a somewhat shifty character. (Russell, 1957)

Donnellan (1966) observed that there is a sense in which Strawson and Russell are both right (and both wrong) about the proper analysis of definite descriptions, because definite descriptions can be used in two different ways. On a so-called *attributive use*, a sentence of the form 'The *F* is a *G*' is used to express a proposition equivalent to 'Whatever is uniquely *F* is a *G*'. Alternatively, on a *referential use*, a sentence of the form 'The *F* is a *G*' is used to pick out a specific individual, *a*, and to say of *a* that *a* is a *G*. Donnellan suggested that Russell's quantificational account of definite descriptions might capture attributive uses, but that it does not work for referential uses. Ludlow in (2007) interprets Donnellan as arguing that in some cases descriptions are Russellian and in other cases they are Strawsonian.

Kripke (1977) responded to Donnellan by arguing that the Russellian account of definite descriptions could, by itself, account for both referential and attributive uses, and that the difference between the two cases could be entirely a matter of pragmatics, because there is an important distinction between what one literally says by an utterance and what one intends to communicate by that utterance. Neale (1990) supported Russell's view by collecting a number of previously observed cases in which intuitions about truth conditions clearly do not support Strawson's view. On the other hand, a number of linguists have recently come to Strawson's defence on this matter. For a detailed survey of the arguments supporting Strawson's view and also arguments supporting Russell's, see (Ludlow, 2007). Here it might suffice to say that Strawson's concerns have not delivered a knock-out blow to Russell's theory of descriptions, and so this topic remains very much alive. Recently, von Stechow (2004) argues that every sentence containing a definite description 'the *F*' comes with the existential presupposition that there be a unique *F*. For instance, he argues against the standpoint that the sentence "Last week, my friend went for a drive with the king of France" is false. He claims that this sentence presupposes that there be a king of France and that in the technical sense the sentence has no truth-value.

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<sup>8</sup> Nevertheless, for Strawson, *sentences* are meaningful in and of themselves, independently of the empirical facts like contingent non-existence of the King of France.

In this chapter I am not going to take into account Kripke's pragmatic factors like the intentions of a speaker. In other words, I am not going to take into account what a speaker might have meant by his/her utterance, for this is irrelevant to a *logical* semantic theory. So I am disregarding Donnellan's troublesome notion of having somebody in mind. Instead, I will propose a *literal semantic analysis* of sentences of the form "The *F* is a *G*". What I want to show is this. First, definite descriptions are not deprived of a self-contained meaning and they denote one and the same entity in any context. Thus they are never Russellian. Second, Russell's insight that a definite description 'the *F*' does not denote a definite individual is spot-on. Rather, according to TIL, 'the *F*' denotes a *condition* to be contingently satisfied by the individual (if any) that happens to be the *F*. I will explicate such conditions in terms of possible-world intensions, *viz.* as individual roles or offices to be occupied by at most one individual per world/time pair. Third, I am going to show that Donnellan was right that sentences of the form "The *F* is a *G*" are ambiguous. However, their ambiguity does not concern a shift of meaning of the definite description 'the *F*'. Rather, the ambiguity concerns different *topic-focus* articulations of these sentences. There are two options. The description 'the *F*' may occur in the topic of a sentence and property *G* (the focus) is predicated of the topic. This case corresponds to Donnellan's *referential* use; using medieval terminology I will say that 'the *F*' occurs with *de re* supposition. The other option is '*G*' occurring as topic and 'the *F*' as focus. This reading corresponds to Donnellan's *attributive* use of 'the *F*' and the description occurs with *de dicto* supposition. Consequently, such sentences are ambiguous between their *de dicto* and *de re* readings. On their *de re* reading they *presuppose* the existence of a unique *F*. Thus Strawson's analysis appears to be adequate for *de re* cases. On their *de dicto* reading they have the truth-conditions as specified by the Russellian analysis. They do not presuppose, but only entail, the existence of a unique *F*. However, the Russellian analysis, though being equivalent to the one I am going to propose, is not an adequate literal analysis of *de dicto* readings.

I am going to bring out the *semantic* nature of the topic-focus difference by means of a logical analysis. As a result, I will furnish sentences differing only in their topic-focus articulation with different structured meanings producing different possible-world propositions.<sup>9</sup> Moreover, the proposed solution of the problem of definite descriptions generalizes to any sentences differing in their topic-focus articulation. Thus I am going to introduce a *general analytic schema* of sentences that come with a presupposition. Since our logic is a hyperintensional logic of *partial functions*, I am able to analyse sentences with presuppositions in a natural way. It means that I furnish them with hyperpropositions, *viz.* procedures that produce partial possible-world propositions, which occasionally yield truth-value gaps.<sup>10</sup>

### 3. Foundations of TIL

TIL is an overarching semantic theory for all sorts of discourse, whether colloquial, scientific, mathematical or logical. The theory is a *procedural* (as opposed to denotational) one, according to which sense is an abstract, extra-linguistic procedure detailing what operations to apply to what procedural constituents to arrive at the product (if any) of the

<sup>9</sup> For details on structured meanings, see (Duží, et al., 2010b).

<sup>10</sup> For an introduction to the notion of hyperproposition, see (Jespersen, 2010).

procedure. Such procedures are rigorously defined as TIL *constructions*. The semantics is tailored to the hardest case, as constituted by hyperintensional contexts, and generalized from there to simpler intensional and extensional contexts. This entirely anti-contextual and compositional semantics is, to the best of my knowledge, the only one that deals with all kinds of context in a uniform way. Thus we can characterize TIL as an extensional logic of hyperintensions.<sup>11</sup> The sense of an empirical sentence is an algorithmically structured *construction* of the proposition denoted by the sentence. The denoted proposition is a flat, or unstructured, mapping with domain in a logical space of possible worlds. Our motive for working ‘top-down’ has to do with anti-contextualism: any given unambiguous term or expression (even one involving indexicals or anaphoric pronouns) expresses the same construction as its sense whatever sort of context the term or expression is embedded within. And the meaning of an expression determines the respective denoted entity (if any), but not vice versa. The denoted entities are (possibly 0-ary) functions understood as set-theoretical mappings. Thus we strictly distinguish between a procedure (construction) and its product (here, a constructed function), and between a function and its value. What makes TIL suitable for the job of disambiguation is the fact that the theory construes the semantic properties of the sense and denotation relations as remaining invariant across different sorts of linguistic contexts.<sup>12</sup> Thus logical analysis disambiguates ambiguous expressions in such a way that an ambiguous expression is furnished with more than one context-invariant meaning that is TIL construction. However, *logical* analysis cannot dictate *which* disambiguation is the intended one. It falls to *pragmatics* to select the intended one.

### 3.1 Semantic foundations of TIL

The context-invariant semantics of TIL is obtained by universalizing Frege’s reference-shifting semantics custom-made for ‘indirect’ contexts.<sup>13</sup> The upshot is that it becomes trivially true that all contexts are transparent, in the sense that pairs of terms that are co-denoting outside an indirect context remain co-denoting inside an indirect context and vice versa. In particular, definite descriptions that only contingently describe the same individual never qualify as co-denoting.<sup>14</sup> Our term for the extra-semantic, factual relation of contingently describing the same entity is ‘*reference*’, whereas ‘*denotation*’ stands for the intra-semantic, pre-factual relation between two words that pick out the same entity at the same world/time-pairs.

The syntax of TIL is Church’s (higher-order) typed  $\lambda$ -calculus, but with the all-important difference that the syntax has been assigned a *procedural* (as opposed to denotational) semantics. Thus, abstraction transforms into the molecular procedure of forming a function, application into the molecular procedure of applying a function to an argument, and variables into atomic procedures for arriving at their values. Furthermore, TIL constructions represent our interpretation of Frege’s notion of *Sinn* (with the exception that constructions are not truth-bearers; instead some present either truth-values or truth-conditions) and are kindred to Church’s notion of *concept*. Constructions are linguistic

<sup>11</sup> For the most recent application, see (Duží & Jespersen, forthcoming).

<sup>12</sup> *Indexicals* being the only exception: while the sense of an indexical remains constant, its denotation varies in keeping with its contextual embedding. See (Duží et al., 2010a, § 3.4).

<sup>13</sup> See (Frege, 1892).

<sup>14</sup> See Definition 7.

senses as well as modes of presentation of objects and are our hyperintensions. While the Frege-Church connection makes it obvious that constructions are not formulae, it is crucial to emphasize that constructions are not functions(-in-extension), either. They might be explicated as Church's 'functions-in-intension', but we do not use the term 'function-in-intension', because Church did not define it (he only characterized functions-in-intension as rules for presenting functions-in-extension). Rather, technically speaking, some constructions are modes of presentation of functions, including 0-place functions such as individuals and truth-values, and the rest are modes of presentation of other constructions. Thus, with constructions of constructions, constructions of functions, functions, and functional values in our stratified ontology, we need to keep track of the traffic between multiple logical strata. The ramified type hierarchy does just that. What is important about this traffic is, first of all, that constructions may themselves figure as functional arguments or values. Thus we consequently need constructions of one order higher in order to present those being arguments or values of functions. With both hyperintensions and possible-world intensions in its ontology, TIL has no trouble assigning either hyperintensions or intensions to variables as their values. However, the technical challenge of operating on constructions requires two (occasionally three) interrelated, non-standard devices. The first is *Trivialization*, which is an atomic construction, whose only constituent part is itself. The second is the function *Sub* (for 'substitution'). (The third is the function *Tr* (for 'Trivialization'), which takes an object to its Trivialization.) We say that Trivialization is *used to mention* other constructions.<sup>15</sup> The point of mentioning a construction is to make it, rather than what it presents, a functional argument. Hence for a construction to be mentioned is for it to be Trivialized; in this way the context is raised up to a hyperintensional level.

Our neo-Fregean semantic schema, which applies to all contexts, is this triangulation:

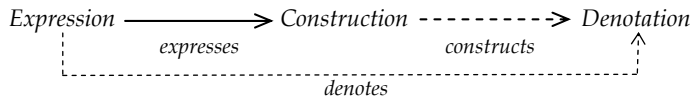


Fig. 1. TIL semantic schema.

The most important relation in this schema is between an expression and its meaning, i.e., a construction. Once *constructions* have been defined, we can logically examine them; we can investigate *a priori* what (if anything) a construction constructs and what is entailed by it. Thus meanings/constructions are semantically primary, denotations secondary, because an expression denotes an object (if any) *via* its meaning that is a construction *expressed* by the expression. Once a construction is explicitly given as a result of logical analysis, the entity (if any) it constructs is already implicitly given. As a limiting case, the logical analysis may reveal that the construction fails to construct anything by being *improper*.

<sup>15</sup> The use/mention distinction normally applies only to *words*; in TIL it applies to the *meanings* of words (i.e., constructions). See (Duži, et al., 2010a, §2.6). In theory, a construction may be mentioned by another construction than Trivialization; but in this chapter we limit ourselves to Trivialization.

### 3.2 Logical foundations of TIL

In this section we set out the definitions of *first-order types* (regimented by a simple type theory), *constructions*, and *higher-order types* (regimented by a ramified type hierarchy), which taken together form the nucleus of TIL, accompanied by some auxiliary definitions.

The type of first-order object includes all objects that are not constructions. Therefore, it includes not only the standard objects of individuals, truth-values, sets, etc., but also functions defined on possible worlds (i.e., the intensions germane to possible-world semantics). Sets, for their part, are always characteristic functions and insofar extensional entities. But the domain of a set may be typed over higher-order objects, in which case the relevant set is itself a higher-order object. Similarly for other functions, including relations, with domain or range in constructions. That is, whenever constructions are involved, we find ourselves in the ramified type hierarchy.<sup>16</sup> The definition of the ramified hierarchy of types decomposes into three parts: firstly, simple types of order 1; secondly, constructions of order  $n$ ; thirdly, types of order  $n + 1$ .

**Definition 1** (*types of order 1*). Let  $B$  be a *base*, where a base is a collection of pair-wise disjoint, non-empty sets. Then:

- i. Every member of  $B$  is an elementary *type of order 1 over B*.
- ii. Let  $\alpha, \beta_1, \dots, \beta_m$  ( $m > 0$ ) be types of order 1 over  $B$ . Then the collection  $(\alpha \beta_1 \dots \beta_m)$  of all  $m$ -ary partial mappings from  $\beta_1 \times \dots \times \beta_m$  into  $\alpha$  is a *functional type of order 1 over B*.

Nothing is a *type of order 1 over B* unless it so follows from (i) and (ii).

**Definition 2** (*construction*)

- i. The *Variable*  $x$  is a *construction* that constructs an object  $X$  of the respective type dependently on a valuation  $v$ ;  $x$  *v-constructs*  $X$ .
- ii. *Trivialization*: Where  $X$  is an object whatsoever (an extension, an intension or a construction),  ${}^0X$  is the *construction Trivialization*. It constructs  $X$  without any change.
- iii. The *Composition*  $[X Y_1 \dots Y_m]$  is the following *construction*. If  $X$  *v-constructs* a function  $f$  of a type  $(\alpha \beta_1 \dots \beta_m)$ , and  $Y_1, \dots, Y_m$  *v-construct* entities  $B_1, \dots, B_m$  of types  $\beta_1, \dots, \beta_m$ , respectively, then the *Composition*  $[X Y_1 \dots Y_m]$  *v-constructs* the value (an entity, if any, of type  $\alpha$ ) of  $f$  on the tuple-argument  $\langle B_1, \dots, B_m \rangle$ . Otherwise the *Composition*  $[X Y_1 \dots Y_m]$  does not *v-construct* anything and so is *v-improper*.
- iv. The *Closure*  $[\lambda x_1 \dots x_m Y]$  is the following *construction*. Let  $x_1, x_2, \dots, x_m$  be pair-wise distinct variables *v-constructing* entities of types  $\beta_1, \dots, \beta_m$  and  $Y$  a construction *v-constructing* an  $\alpha$ -entity. Then  $[\lambda x_1 \dots x_m Y]$  is the *construction  $\lambda$ -Closure* (or *Closure*). It *v-constructs* the following function  $f$  of the type  $(\alpha \beta_1 \dots \beta_m)$ . Let  $v(B_1/x_1, \dots, B_m/x_m)$  be a valuation identical with  $v$  at least up to assigning objects  $B_1/\beta_1, \dots, B_m/\beta_m$  to variables  $x_1, \dots, x_m$ . If  $Y$  is  $v(B_1/x_1, \dots, B_m/x_m)$ -improper (see iii), then  $f$  is undefined on the argument  $\langle B_1, \dots, B_m \rangle$ . Otherwise the value of  $f$  on  $\langle B_1, \dots, B_m \rangle$  is the  $\alpha$ -entity  $v(B_1/x_1, \dots, B_m/x_m)$ -constructed by  $Y$ .

<sup>16</sup> Attempting to type constructions within the simple type theory (as though constructions were first-order objects) is the source of some misconceptions of TIL found in (Daley 2010).



- v. The *Single Execution*  ${}^1X$  is the *construction* that either  $v$ -constructs the entity  $v$ -constructed by  $X$  or, if  $X$   $v$ -constructs nothing, is  *$v$ -improper* (yielding nothing relative to  $v$ ).
- vi. The *Double Execution*  ${}^2X$  is the following *construction*. Where  $X$  is any entity, the *Double Execution*  ${}^2X$  is  *$v$ -improper* (yielding nothing relative to  $v$ ) if  $X$  is not itself a construction, or if  $X$  does not  $v$ -construct a construction, or if  $X$   $v$ -constructs a  $v$ -improper construction. Otherwise, let  $X$   $v$ -construct a construction  $Y$  and  $Y$   $v$ -construct an entity  $Z$ : then  ${}^2X$   $v$ -constructs  $Z$ .

Nothing is a *construction*, unless it so follows from (i) through (vi).

**Definition 3** (ramified hierarchy of types)

$\mathbf{T}_1$  (*types of order 1*). See Definition 1.

$\mathbf{C}_n$  (*constructions of order  $n$* )

- i. Let  $x$  be a variable ranging over a type of order  $n$ . Then  $x$  is a *construction of order  $n$  over  $B$* .
- ii. Let  $X$  be a member of a type of order  $n$ . Then  ${}^0X, {}^1X, {}^2X$  are *constructions of order  $n$  over  $B$* .
- iii. Let  $X, X_1, \dots, X_m$  ( $m > 0$ ) be constructions of order  $n$  over  $B$ . Then  $[X X_1 \dots X_m]$  is a *construction of order  $n$  over  $B$* .
- iv. Let  $x_1, \dots, x_m, X$  ( $m > 0$ ) be constructions of order  $n$  over  $B$ . Then  $[\lambda x_1 \dots x_m X]$  is a *construction of order  $n$  over  $B$* .
- v. Nothing is a *construction of order  $n$  over  $B$*  unless it so follows from  $\mathbf{C}_n$  (i)-(iv).

$\mathbf{T}_{n+1}$  (*types of order  $n + 1$* ). Let  $*_n$  be the collection of all constructions of order  $n$  over  $B$ . Then

- i.  $*_n$  and every type of order  $n$  are *types of order  $n + 1$* .
- ii. If  $0 < m$  and  $\alpha, \beta_1, \dots, \beta_m$  are types of order  $n + 1$  over  $B$ , then  $(\alpha \beta_1 \dots \beta_m)$  (see  $\mathbf{T}_1$  ii)) is a *type of order  $n + 1$  over  $B$* .

Nothing is a *type of order  $n + 1$  over  $B$*  unless it so follows from  $\mathbf{T}_{n+1}$  (i) and (ii).

*Remark.* For the purposes of natural-language analysis, we are currently assuming the following base of ground types, which is part of the ontological commitments of TIL:

- o: the set of truth-values  $\{\mathbf{T}, \mathbf{F}\}$ ;
- t: the set of individuals (the universe of discourse);
- $\tau$ : the set of real numbers (doubling as discrete times);
- $\omega$ : the set of logically possible worlds (the logical space).

Empirical languages incorporate an element of *contingency*, because they denote *empirical conditions* that may or may not be satisfied at some world/time pair of evaluation. Non-empirical languages (in particular the language of mathematics) have no need for an additional category of expressions for empirical conditions. We model these empirical conditions as *possible-world intensions*. They are entities of type  $(\beta\omega)$ : mappings from possible worlds to an arbitrary type  $\beta$ . The type  $\beta$  is frequently the type of the *chronology* of  $\alpha$ -objects, i.e., a mapping of type  $(\alpha\tau)$ . Thus  $\alpha$ -intensions are frequently functions of type  $((\alpha\tau)\omega)$ , abbreviated as ' $\alpha_{\tau\omega}$ '. *Extensional entities* are entities of a type  $\alpha$  where  $\alpha \neq (\beta\omega)$  for any type  $\beta$ . *Examples* of frequently used intensions are: *propositions* of type  $o_{\tau\omega}$ , *properties of individuals* of type  $(o)_{\tau\omega}$ , *binary relations-in-intension* between individuals of type  $(o)_{\tau\omega}$ , *individual*

*offices/roles* of type  $\iota_{\tau\omega}$ . Our *explicit intensionalization and temporalization* enables us to encode constructions of possible-world intensions, by means of terms for possible-world variables and times, directly in the logical syntax. Where variable  $w$  ranges over possible worlds (type  $\omega$ ) and  $t$  over times (type  $\tau$ ), the following logical form essentially characterizes the logical syntax of any empirical language:  $\lambda w \lambda t [\dots w \dots t \dots]$ . Where  $\alpha$  is the type of the object  $v$ -constructed by  $[\dots w \dots t \dots]$ , by abstracting over the values of variables  $w$  and  $t$  we construct a function from worlds to a partial function from times to  $\alpha$ , that is a function of type  $((\alpha\tau)\omega)$ , or ' $\alpha_{\tau\omega}$ ' for short.

Logical objects like *truth-functions* and *quantifiers* are extensional:  $\wedge$  (conjunction),  $\vee$  (disjunction) and  $\supset$  (implication) of type (ooo), and  $\neg$  (negation) of type (oo). The *quantifiers*  $\forall^\alpha$ ,  $\exists^\alpha$  are type-theoretically polymorphous functions of type  $(o(o\alpha))$ , for an arbitrary type  $\alpha$ , defined as follows. The *universal quantifier*  $\forall^\alpha$  is a function that associates a class  $A$  of  $\alpha$ -elements with **T** if  $A$  contains all elements of the type  $\alpha$ , otherwise with **F**. The *existential quantifier*  $\exists^\alpha$  is a function that associates a class  $A$  of  $\alpha$ -elements with **T** if  $A$  is a non-empty class, otherwise with **F**. Another kind of partial polymorphic function we need is the *Singularizer*  $I^\alpha$  of type  $(\alpha(o\alpha))$ . A singularizer is a function that associates a singleton  $S$  with the only member of  $S$ , and is otherwise (i.e. if  $S$  is an empty set or a multi-element set) undefined.

Below all type indications will be provided outside the formulae in order not to clutter the notation. Furthermore, ' $X/\alpha$ ' means that an object  $X$  is (a member) of type  $\alpha$ . ' $X \rightarrow_v \alpha$ ' means that the type of the object  $v$ -constructed by  $X$  is  $\alpha$ . We write ' $X \rightarrow \alpha$ ' if what is  $v$ -constructed does not depend on a valuation  $v$ . This holds throughout:  $w \rightarrow_v \omega$  and  $t \rightarrow_v \tau$ . If  $C \rightarrow_v \alpha_{\tau\omega}$  then the frequently used Composition  $[[C w] t]$ , which is the intensional descent (a.k.a. extensionalization) of the  $\alpha$ -intension  $v$ -constructed by  $C$ , will be encoded as ' $C_{wt}$ '. When using constructions of truth-functions, we often omit Trivialisation and use infix notation to conform to standard notation in the interest of better readability. Also when using constructions of identities of  $\alpha$ -entities,  $=_\alpha/(o\alpha\alpha)$ , we omit Trivialization, the type subscript, and use infix notion when no confusion can arise. For instance, instead of

$$'[{}^0\supset [{}^0=_{\iota} a b] [{}^0=_{((o\tau)\omega)} \lambda w \lambda t [P_{wt} a] \lambda w \lambda t [P_{wt} b]]]'$$

where  $=_{\iota}/(o\iota\iota)$  is the identity of individuals and  $=_{((o\tau)\omega)}/(oo_{\tau\omega}o_{\tau\omega})$  the identity of propositions;  $a, b$  constructing objects of type  $\iota$ ,  $P$  objects of type  $(o\iota)_{\tau\omega}$  we write

$$'[[a = b] \supset [\lambda w \lambda t [P_{wt} a] = \lambda w \lambda t [P_{wt} b]]]'$$

We invariably furnish expressions with procedural structured meanings, which are explicated as TIL constructions. The analysis of an unambiguous sentence thus consists in discovering the logical construction encoded by a given sentence. The *TIL method of analysis* consists of three steps:

1. *Type-theoretical analysis*, i.e., assigning types to the objects that receive mention in the analysed sentence.
2. *Type-theoretical synthesis*, i.e., combining the constructions of the objects *ad* (1) in order to construct the proposition of type  $o_{\tau\omega}$  denoted by the whole sentence.
3. *Type-theoretical checking*, i.e. checking whether the proposed analysans is type-theoretically coherent.

To illustrate the method, we analyse the notorious sentence “The King of France is bald” in the Strawsonian way. The sentence talks about the office of the King of France (topic) ascribing to the individual (if any) that occupies this office the property of being bald (focus). Thus it is presupposed that the King of France exist, i.e., that the office be occupied. If it is not, then the proposition denoted by the sentence has no truth-value.<sup>17</sup> This fact has to be revealed by our analysis. Here is how.

*Ad (1) King\_of/(ι)<sub>τω</sub>*: an empirical function that dependently on  $\langle w, t \rangle$ -pairs assigns to one individual (a country) another individual (its king); *France/ι*; *King\_of\_France/ι<sub>τω</sub>*; *Bald/(οι)<sub>τω</sub>*.

*Ad (2) and (3)*. For the sake of simplicity, I will demonstrate the steps (2) and (3) simultaneously. In the second step we combine the *constructions* of the objects *ad (1)* in order to construct the proposition (of type  $ο_{τω}$ ) denoted by the whole sentence. Since we intend to arrive at the *literal* analysis of the sentence, the objects denoted by the semantically simple expressions are constructed by their Trivialisations:  ${}^0King\_of$ ,  ${}^0France$ ,  ${}^0Bald$ . In order to construct the office *King\_of\_France*, we have to combine  ${}^0King\_of$  and  ${}^0France$ . The function *King\_of* must be extensionalised first *via* the Composition  ${}^0King\_of_{wt} \rightarrow_v (ι)$ , and the result is then applied to *France*; we get  $[{}^0King\_of_{wt} {}^0France] \rightarrow_v ι$ . Abstracting over the values of  $w$  and  $t$  we obtain the Closure that constructs the office:  $λwλt [{}^0King\_of_{wt} {}^0France] \rightarrow ι_{τω}$ . But the property of being bald cannot be ascribed to an individual office. Rather, it is ascribed to the individual (if any) occupying the office. Thus the office has to be subjected to intensional descent first:  $λwλt [{}^0King\_of_{wt} {}^0France]_{wt} \rightarrow_v ι$ . The property itself has to be extensionalised as well:  ${}^0Bald_{wt}$ . By Composing these two constructions, we obtain either a truth-value (**T** or **F**) or nothing, according as the King of France is, or is not, bald, or does not exist, respectively. Finally, by abstracting over the values of the variables  $w$  and  $t$ , we construct the proposition:

$$λwλt [{}^0Bald_{wt} λwλt [{}^0King\_of_{wt} {}^0France]_{wt}]$$

*Gloss*. In *any* world ( $λw$ ) at *any* time ( $λt$ ) do this. First, find out who is the King of France:  $[{}^0King\_of_{wt} {}^0France]$ . If there is none, then terminate with a truth-value gap because the Composition  $[{}^0King\_of_{wt} {}^0France]$  is  $v$ -improper. Otherwise, check whether the so obtained individual has the property of being bald:  $[{}^0Bald_{wt} [{}^0King\_of_{wt} {}^0France]]$ . If he is, then **T**, otherwise **F**. So much for the method of analysis and the semantic schema of the logic of TIL.

#### 4. Definite descriptions: Strawsonian or Russellian?

Now I am going to propose a solution to the Strawson-Russell standoff. In other words, I am going to analyse the phenomena of presupposition and entailment connected with using definite descriptions with supposition *de dicto* or *de re*, and I will show how the topic-focus distinction determines which of the two cases applies.

<sup>17</sup> On our approach this does not mean that the sentence is meaningless. The sentence has a sense, namely an instruction of how to evaluate in any possible world  $w$  at any time  $t$  its truth-conditions. (Such instructions are encoded in our language of constructions.) Only if we evaluate these conditions in such a state-of-affairs where there is no King of France does the process of evaluation yield a truth-value gap.

#### 4.1 Topic-focus ambiguity

When used in a communicative act, a sentence communicates something (the focus  $F$ ) about something (the topic  $T$ ). Thus the schematic structure of a sentence is  $F(T)$ . The topic  $T$  of a sentence  $S$  is often associated with a presupposition  $P$  of  $S$  such that  $P$  is entailed both by  $S$  and  $\text{non-}S$ . On the other hand, the clause in the focus usually occasions a mere entailment of some  $P$  by  $S$ . To give an example, consider the sentence “Our defeat was caused by John”.<sup>18</sup> There are two possible readings of this sentence. Taken one way, the sentence is about our defeat, conveying the snippet of information that it was caused by John. In such a situation the sentence is associated with the presupposition that we were defeated. Indeed, the negated form of the sentence, “Our defeat was not caused by John”, also implies that we were defeated. Thus ‘our defeat’ is the topic and ‘was caused by John’ the focus clause. Taken the other way, the sentence is about the topic John, ascribing to him the property that he caused our defeat (focus). Now the scenario of truly asserting the negated sentence can be, for instance, the following. Though it is true that John has a reputation for being rather a bad player, Paul was in excellent shape and so we won. Or, another scenario is thinkable. We were defeated, only not because of John but because the whole team performed badly. Hence, our being defeated is not presupposed by this reading, it is only entailed.

Schematically, if  $\models$  is the relation of entailment, then the logical difference between a mere entailment and a presupposition is this:

$P$  is a *presupposition* of  $S$ :  $(S \models P)$  and  $(\text{non-}S \models P)$

*Corollary*: If  $P$  is not true, then *neither*  $S$  *nor*  $\text{non-}S$  is true. Hence,  $S$  has no truth-value.

$P$  is only *entailed* by  $S$ :  $(S \models P)$  and neither  $(\text{non-}S \models P)$  nor  $(\text{non-}S \models \text{non-}P)$

*Corollary*: If  $S$  is not true, then we cannot deduce anything about the truth-value of  $P$ .

More precisely, the entailment relation obtains between hyperpropositions  $P, S$ ; i.e., the *meaning* of  $P$  is analytically entailed or presupposed by the *meaning* of  $S$ . Thus  $\models / ((o^* *_n)^*)$  is defined as follows. Let  $C^S, C^P$  be constructions assigned to sentences  $S, P$ , respectively, as their meanings. Then  $S$  *entails*  $P$  ( $C^S \models C^P$ ) iff the following holds:<sup>19</sup>

$$\forall w \forall t \ [ [{}^0 \text{True}_{wt} C] \supset [{}^0 \text{True}_{wt} C'] ]$$

Since we work with properly *partial* functions, we need to apply the propositional property  $\text{True} / (oo_{\tau\omega})_{\tau\omega}$ , which returns **T** for those  $\langle w, t \rangle$ -pairs at which the argument proposition is true, and **F** in all the remaining cases. There are two other propositional properties: *False* and *Undef*, both of type  $(oo_{\tau\omega})_{\tau\omega}$ . The three properties are defined as follows. Let  $P$  be a propositional construction ( $P / *_n \rightarrow o_{\tau\omega}$ ). Then

$[{}^0 \text{True}_{wt} P]$  *v*-constructs the truth-value **T** iff  $P_{wt}$  *v*-constructs **T**, otherwise **F**.

$[{}^0 \text{False}_{wt} P]$  *v*-constructs the truth-value **T** iff  $[\neg P_{wt}]$  *v*-constructs **T**, otherwise **F**.

$[{}^0 \text{Undef}_{wt} P]$  *v*-constructs the truth-value **T** iff

$$[\neg [{}^0 \text{True}_{wt} P] \wedge \neg [{}^0 \text{False}_{wt} P]] \text{ *v*-constructs **T**, otherwise **F** .}$$

<sup>18</sup> This and some other examples were taken from Hajičová (2008).

<sup>19</sup> For the general definition of entailment and the difference between analytical and logical entailment, see (Duží 2010).

Thus we have:

$$\begin{aligned}\neg[{}^0\text{Undef}_{wt} P] &= [[{}^0\text{True}_{wt} P] \vee [{}^0\text{False}_{wt} P]] \\ \neg[{}^0\text{True}_{wt} P] &= [[{}^0\text{False}_{wt} P] \vee [{}^0\text{Undef}_{wt} P]] \\ \neg[{}^0\text{False}_{wt} P] &= [[{}^0\text{True}_{wt} P] \vee [{}^0\text{Undef}_{wt} P]]\end{aligned}$$

Hence, though we work with truth-value gaps, we do not work with a third truth-value, and our logic is in this weak sense bivalent.

## 4.2 The King of France revisited

Above we analysed the sentence “The King of France is bald” on its perhaps most natural reading as predicating the property of being bald (the focus) of the individual (if any) that is the present King of France (the topic). Yet there is another, albeit less natural reading of the sentence. Imagine that the sentence is uttered in a situation when we are talking about baldness, and somebody asks “Who is bald?” The answer might be “Well, among those who are bald there is the present King of France”. If you got such an answer, you would most probably protest, “This cannot be true, for there is no King of France now”. On such a reading the sentence is about baldness (topic) claiming that this property is instantiated, among others, by the King of France (focus). Since there are no rigorous grammatical rules in English to distinguish between the two variants, the input of our *logical* analysis is the result of a *linguistic* analysis, where the topic and focus of a sentence are made explicit.<sup>20</sup> In this chapter I will mark the topic clause in italics. The two readings of the above sentence are:

- (S)     *“The King of France is bald”* (Strawsonian) and  
(R)     “*The King of France is bald*” (Russellian).

The analysis of (S) is as above:

$$\lambda w \lambda t [{}^0\text{Bald}_{wt} \lambda w \lambda t [{}^0\text{King\_of}_{wt} {}^0\text{France}]_{wt}]$$

The meaning of ‘the King of France’, *viz.*  $\lambda w \lambda t [{}^0\text{King\_of}_{wt} {}^0\text{France}]$ , occurs in (S) with *de re* supposition, because the object of predication is the unique *value* in a  $\langle w, t \rangle$ -pair of evaluation of the office rather than the office itself.<sup>21</sup> The following *two de re principles* are satisfied: the principle of *existential presupposition* and the principle of *substitution of co-referential* expressions. Thus the following arguments are valid (though not sound):

*The King of France is/is not bald*  
The King of France exists

<sup>20</sup> For instance, the Prague linguistic school created The Prague Dependency Treebank for the Czech language, which contains a large amount of Czech texts with complex and interlink annotation on different levels. The tectogrammatical representation contains the semantic structure of sentences with topic-focus annotators. For details, see <http://ufal.mff.cuni.cz/pdt2.0/>.

<sup>21</sup> For details on *de dicto* vs. *de re* supposition, see (Duží *et al.*, 2010a), esp. §§ 1.5.2 and 2.6.2, and also (Duží 2004).

*The King of France is bald*  
*The King of France is Louis XVI*  
*Louis XVI is bald*

Here are the proofs.

(a) *existential presupposition*:

First, existence is here a property of an individual *office* rather than of some non-existing individual (whatever it might mean for an individual not to exist). Thus we have  $Exist/(o_{\tau\omega})_{\tau\omega}$ . To prove the validity of the first argument, we define  $Exist/(o_{\tau\omega})_{\tau\omega}$  as the property of an office's being occupied at a given world/time pair:

$${}^0Exist =_{of} \lambda w \lambda t \lambda c [{}^0\exists \lambda x [x =_i c_{wt}]],$$

i. e.  $[{}^0Exist_{wt} c] =_o [{}^0\exists \lambda x [x =_i c_{wt}]]$

*Types*:  $\exists/(o(o))$ : the class of non-empty classes of individuals;  $c \rightarrow_v \iota_{\tau\omega}$ ;  $x \rightarrow_v \iota$ ;  $=_o/(ooo)$ : the identity of truth-values;  $=_{of}/(o(o_{\tau\omega})_{\tau\omega}(o_{\tau\omega})_{\tau\omega})$ : the identity of properties of individual offices;  $=_i/(oi)$ : the identity of individuals,  $x \rightarrow_v \iota$ . Now let *Louis*/ $\iota$ , *Empty*/ $(o(o))$  the singleton containing the empty set of individuals, and *Improper*/ $(o*1)_{\tau\omega}$  the property of constructions of being  $v$ -improper at a given  $\langle w, t \rangle$ -pair, the other types as above. Then at any  $\langle w, t \rangle$ -pair the following proof steps are truth-preserving:

- |    |   |                           |
|----|---|---------------------------|
| 1) | ( $\neg$ )[ ${}^0Bald_{wt} \lambda w \lambda t [{}^0King\_of_{wt} {}^0France]_{wt}$ ]             | $\emptyset$               |
| 2) | $\neg[{}^0Improper_{wt} [{}^0\lambda w \lambda t [{}^0King\_of_{wt} {}^0France]_{wt}]]$           | by Def. 2, iii)           |
| 3) | $\neg[{}^0Empty \lambda x [x =_i [{}^0\lambda w \lambda t [{}^0King\_of_{wt} {}^0France]_{wt}]]]$ | from (2) by Def. 2, iv)   |
| 4) | $[{}^0\exists \lambda x [x =_i [{}^0\lambda w \lambda t [{}^0King\_of_{wt} {}^0France]_{wt}]]]$   | EG                        |
| 7) | $[{}^0Exist_{wt} [{}^0\lambda w \lambda t [{}^0King\_of_{wt} {}^0France]_{wt}]]]$                 | by def. of <i>Exist</i> . |

(b) *substitution*:

- |    |   |                            |
|----|---|----------------------------|
| 1) | [ ${}^0Bald_{wt} \lambda w \lambda t [{}^0King\_of_{wt} {}^0France]_{wt}$ ] | $\emptyset$                |
| 2) | [ ${}^0Louis =_i \lambda w \lambda t [{}^0King\_of_{wt} {}^0France]_{wt}$ ] | $\emptyset$                |
| 3) | [ ${}^0Bald_{wt} {}^0Louis$ ]   | substitution of identicals |

As explained above, the sentence (R) is not associated with the presupposition that the present King of France exist, because 'the King of France' occurs now in the focus clause. The truth-conditions of the Russellian "The King of France is *bald*" are these:

- True, if among those who are bald there is the King of France
- False, if among those who are bald there is no King of France (either because the present King of France does not exist or because the King of France is not bald).

Thus the two readings (S) and (R) have different truth-conditions, and they are not equivalent, albeit they are co-entailing. The reason is this. Trivially, a valid argument is *truth-preserving from premises to conclusion*. However, due to partiality, the entailment relation may fail to be *falsity-preserving from conclusion to premises*. As a consequence, if  $A, B$  are constructions of propositions such that  $A \models B$  and  $B \models A$ , then  $A, B$  are not necessarily equivalent in the sense of constructing the same proposition. The propositions they construct may not be identical, though the propositions take the truth-value T at exactly the same world/times, because they may differ in such a way that at some  $\langle w, t \rangle$ -pair(s) one takes the value F while the other is

undefined. The pair of meanings of (S) and (R) is an example of such co-entailing, yet non-equivalent hyperpropositions. If the value of the proposition constructed by the meaning of (S) is **T** then so is the value of the proposition constructed by the meaning of (R), and *vice versa*. But, for instance, in the actual world now the proposition constructed by (S) has *no truth-value* whereas the proposition constructed by (R) takes value **F**.

Now I am going to analyse (R). Russell argued for his theory in (1905, p. 3):

The evidence for the above theory is derived from the difficulties which seem unavoidable if we regard denoting phrases as standing for genuine constituents of the propositions in whose verbal expressions they occur. Of the possible theories which admit such constituents the simplest is that of Meinong. This theory regards any grammatically correct denoting phrase as standing for an object. Thus 'the present King of France', 'the round square', etc., are supposed to be genuine objects. It is admitted that such objects do not subsist, but nevertheless they are supposed to be objects. This is in itself a difficult view; but the chief objection is that such objects, admittedly, are apt to infringe the law of contradiction. It is contended, for example, that the existent present King of France exists, and also does not exist; that the round square is round, and also not round, etc. But this is intolerable; and if any theory can be found to avoid this result, it is surely to be preferred.

We have such a theory at hand, *viz.* TIL. Moreover, TIL makes it possible to avoid the other objections against Russell's analysis as well. Russellian rephrasing of the sentence "The King of France is *bald*" is this: "There is a unique individual such that he is the King of France and he is bald". This sentence expresses the construction

$$(R^*) \quad \lambda w \lambda t [{}^0 \exists \lambda x [x =_i [\lambda w \lambda t [{}^0 \text{King\_of}_{wt} {}^0 \text{France}]_{wt}] \wedge [{}^0 \text{Bald}_{wt} x]]].^{22}$$

TIL analysis of the 'Russellian rephrasing' does not deprive 'the King of France' of its meaning. The meaning is invariably, in all contexts, the Closure  $\lambda w \lambda t [{}^0 \text{King\_of}_{wt} {}^0 \text{France}]$ . Thus the second objection to the Russellian analysis is not pertinent here. Moreover, even the third objection is irrelevant, because in  $(R^*) \lambda w \lambda t [{}^0 \text{King\_of}_{wt} {}^0 \text{France}]$  occurs intensionally unlike in the analysis of (S) where it occurs extensionally.<sup>23</sup> The existential quantifier  $\exists$  applies to *sets* of individuals rather than a particular individual. The proposition constructed by  $(R^*)$  is true if the *set* of individuals who are bald contains the individual who occupies the office of King of France, otherwise it is simply false. The truth conditions specified by  $(R^*)$  are Russellian. Thus we might be content with  $(R^*)$  as an adequate analysis of the Russellian reading (R). Yet we should not be. The reason is this. Russell's analysis has another defect; it does not comply with *Carnap's principle of subject-matter*, which states, roughly, that only those entities that receive mention in a sentence can become constituents of its meaning.<sup>24</sup> In

<sup>22</sup> Note that in TIL we do not need the construction corresponding to  $\forall y (Fy \supset x=y)$  specifying the uniqueness of the King of France, because it is inherent in the meaning of 'the King of France'. This holds also in a language like Czech, which lacks grammatical articles. The meaning of descriptions 'the King of France', 'král Francie' is a construction of an individual office of type  $\iota_{\omega}$  occupied in each  $\langle w, t \rangle$ -pair by at most one individual.

<sup>23</sup> For the definition of extensional, intensional and hyperintensional occurrence of a construction, see (Duží et al., 2010a, § 2.6).

<sup>24</sup> See (Carnap 1947, §24.2, §26).

other words, ( $R^*$ ) is not the literal analysis of the sentence “The King of France is *bald*”., because existence and conjunction do not receive mention in the sentence. Russell did avoid the intolerable result that the King of France both does and does not exist, but the price he paid is too high, because his rephrasing of the sentence is too loose a reformulation of it. TIL, as a hyperintensional, typed *partial*  $\lambda$ -calculus, is in a much better position to solve the problem.

From the logical point of view, the two readings differ in the way their respective *negated* form is obtained. Whereas the Strawsonian negated form is “The *King of France* is *not bald*”, which obviously lacks a truth-value if the King of France does not exist, the Russellian negated form is “It is not true that the King of France is *bald*”, which is true at those  $\langle w, t \rangle$ -pairs where the office is not occupied. Thus in the Strawsonian case the property of not being bald is ascribed to the individual, if any, that occupies the royal office. The meaning of ‘the King of France’ occurs with *de re* supposition, as we have seen above. On the other hand, in the Russellian case the property of not being true is ascribed to the whole proposition that the King is bald, and thus (the same meaning of) the description ‘the King of France’ occurs with *de dicto* supposition. Hence we simply ascribe the property of being or not being true to the whole proposition. To this end we apply the propositional property  $True / (oo_{\tau\omega})_{\tau\omega}$  defined above. Now the analysis of the sentence (R) is this construction:

$$(R') \quad \lambda w \lambda t [{}^0True_{wt} \lambda w \lambda t [{}^0Bald_{wt} \lambda w \lambda t [{}^0King\_of_{wt} {}^0France]_{wt}]]$$

Neither ( $R'$ ) nor its negation

$$(R'_{neg}) \quad \lambda w \lambda t \neg [{}^0True_{wt} \lambda w \lambda t [{}^0Bald_{wt} \lambda w \lambda t [{}^0King\_of_{wt} {}^0France]_{wt}]]$$

entail that the King of France exists, which is just as it should be. ( $R'_{neg}$ ) constructs the proposition *non-P* that takes the truth-value T if the proposition that the King of France is bald takes the value F (because the King of France is not bald) or is undefined (because the King of France does not exist).

Consider now another group of sample sentences:

- (1) “The King of France visited London yesterday.”  
 (1') “The King of France did not visit London yesterday.”

The sentences (1) and (1') talk about the (actual and current) King of France (the topic), ascribing to him the property of (not) having visited London yesterday (the focus). Thus both sentences share the presupposition that the King of France actually exist *now*. If this presupposition fails to be satisfied, then neither of the propositions expressed by (1) and (1') has a truth-value. The situation is different in the case of sentences (2) and (2'):

- (2) “London was visited by the King of France yesterday.”  
 (2') “London was not visited by the King of France yesterday.”

Now the property (the focus) of having been visited by the King of France yesterday is predicated of London (the topic). The existence of the King of France (now) is presupposed neither by (2) nor by (2'). The sentences can be read as “Among the visitors of London yesterday was (not) the King of France”. The existence of the King of France *yesterday* is only



entailed by (2) and *not presupposed*.<sup>25</sup> Our analyses respect these conditions. Let *Yesterday*/ $((\sigma\tau)\tau)$  be the function that associates a given time  $t$  with the time interval that is yesterday with respect to  $t$ ; *Visit*/ $(\text{ou})_{\tau\omega}$ ; *King\_of*/ $(\text{u})_{\tau\omega}$ ; *France*/ $v$ ;  $\exists^*/(\text{o}(\sigma\tau))$ : the existential quantifier that assigns to a given set of times the truth-value **T** if the set is non-empty, otherwise **F**. In what follows I will use an abbreviated notation without Trivialisation, writing ' $\exists x A$ ' instead of ' $[\text{O}\exists^*\lambda x A]$ ', when no confusion can arise. The analyses of sentences (1), (1') come down to

- (1\*)  $\lambda w \lambda t [\lambda x \exists t' [[[\text{O} \text{Yesterday } t] t'] \wedge [\text{O} \text{Visit}_{wt'} x \text{O} \text{London}]] \lambda w \lambda t [\text{O} \text{King\_of}_{wt} \text{O} \text{France}]_{wt}$   
 (1'\*)  $\lambda w \lambda t [\lambda x [\exists t' [[[\text{O} \text{Yesterday } t] t'] \wedge \neg [\text{O} \text{Visit}_{wt'} x \text{O} \text{London}]]] \lambda w \lambda t [\text{O} \text{King\_of}_{wt} \text{O} \text{France}]_{wt}$

At such a  $\langle w, t \rangle$ -pair at which the King of France does not exist neither of the propositions constructed by (1\*) and (1'\*) has a truth-value, because the extensionalization of the office yields no individual, the Composition  $\lambda w \lambda t [\text{O} \text{King\_of}_{wt} \text{O} \text{France}]_{wt}$  being  $v$ -improper. We have the Strawsonian case, the meaning of 'King of France' occurring with *de re* supposition, and the King's existence being presupposed. On the other hand, the sentences (2), (2') express

- (2\*)  $\lambda w \lambda t \exists t' [[[\text{O} \text{Yesterday } t] t'] \wedge [\text{O} \text{Visit}_{wt'} \lambda w \lambda t [\text{O} \text{King\_of}_{wt} \text{O} \text{France}]_{wt'} \text{O} \text{London}]]$   
 (2'\*)  $\lambda w \lambda t \exists t' [[[\text{O} \text{Yesterday } t] t'] \wedge \neg [\text{O} \text{Visit}_{wt'} \lambda w \lambda t [\text{O} \text{King\_of}_{wt} \text{O} \text{France}]_{wt'} \text{O} \text{London}]]$

At such a  $\langle w, t \rangle$ -pair at which the proposition constructed by (2\*) is true, the Composition  $\exists t' [[[\text{O} \text{Yesterday } t] t'] \wedge \lambda w \lambda t [\text{O} \text{King\_of}_{wt} \text{O} \text{France}]_{wt'} \text{O} \text{London}]]$   $v$ -constructs **T**. This means that the second conjunct  $v$ -constructs **T** as well and the Composition  $\lambda w \lambda t [\text{O} \text{King\_of}_{wt} \text{O} \text{France}]_{wt'}$  is not  $v$ -improper. Thus the King of France *existed at some time  $t'$  belonging to yesterday*. On the other hand, if the King of France did not exist at any time yesterday, then the Composition  $\lambda w \lambda t [\text{O} \text{King\_of}_{wt} \text{O} \text{France}]_{wt'}$  is  $v$ -improper for any  $t'$  belonging to yesterday and the time interval  $v$ -constructed by  $\lambda t' [[[\text{O} \text{Yesterday } t] t'] \wedge [\text{O} \text{Visit}_{wt'} \lambda w \lambda t [\text{O} \text{King\_of}_{wt} \text{O} \text{France}]_{wt'} \text{O} \text{London}]]$ , as well as by  $\lambda t' [[[\text{O} \text{Yesterday } t] t'] \wedge \neg [\text{O} \text{Visit}_{wt'} \lambda w \lambda t [\text{O} \text{King\_of}_{wt} \text{O} \text{France}]_{wt'} \text{O} \text{London}]]$ , is empty. The existential quantifier takes this interval to **F**. This is as it should be, because (2\*) *only implies the existence* of the King of France *yesterday* but *does not presuppose* it. We have the Russellian case. The meaning of the definite description 'the King of France' occurs with *de dicto* supposition in (2) and (2').<sup>26</sup>

## 5. Topic-focus ambivalence in general

Up until now we have utilised the singularity of definite descriptions like 'the King of France' that denote functions of type  $\iota_{\tau\omega}$ . If the King of France does not exist in some particular world  $W$  at some particular time  $T$ , the office is not occupied and the function does not have a value at  $\langle W, T \rangle$ . Due to the partiality of the office constructed by  $\lambda w \lambda t [\text{O} \text{King\_of}_{wt} \text{O} \text{France}]$  and the principle of compositionality, the respective analyses construct purely partial propositions associated with some presupposition, as desired. Now I am going to generalize the topic-focus phenomenon to sentences containing general terms.

<sup>25</sup> Von Stechow (2004) does not take into account this reading and says that any sentence containing 'the King of France' comes with the presupposition that the King of France exist *now*. In my opinion, this is because he considers only the *neutral* reading, thus rejecting topic-focus ambiguities.

<sup>26</sup> More precisely, the meaning of 'the King of France' occurs with *de dicto* supposition with respect to the temporal parameter  $t$ .

To get started, let us analyse Strawson's example

- (3) "All John's children are asleep."  
 (3') "All John's children are not asleep."

According to Strawson both (1) and (1') entail<sup>27</sup>

- (4) John has children.

In other words, (4) is a presupposition of (3) and (3'). If each of John's children is asleep, then (3) is true and (3') false. If each of John's children is not asleep, then (3) is false and (3') is true. However, if John has no children, then (3) and (3') are neither true nor false. Note that applying a classical regimentation of (3) in the language of the first-order predicate logic (FOL), we get

$$"\forall x [JC(x) \supset S(x)]"$$

This formula is true under every interpretation assigning an empty set of individuals to the predicate *JC* ('is a child of John's'). In other words, FOL does not make it possible to render the truth-conditions of a sentence equipped with a presupposition, because FOL is a logic of *total* functions. We need to apply a richer logical system in order to express the instruction how to evaluate the truth-conditions of (3) in the way described above. By reformulating the above specification of the truth-conditions of (3) in a rather technical jargon of English, we get

"If John has children *then* check whether all his children are asleep,  
*else* fail to produce a truth-value."

We now analyse the particular constituents of this instruction. As always, we start with assigning types to the objects that receive mention in the sentence: *Child\_of*((o)t)<sub>τω</sub>: an empirical function that dependently on states-of-affairs assigns to an individual a set of individuals, its children; *John*/v; *Sleep*/(o)τω; ∃/(o(o)); *All*/((o(o)))(o): a restricted general quantifier that assigns to a given set the set of all its supersets.

The presupposition that John have children receives the analysis

$$\lambda w \lambda t [{}^0 \exists \lambda x [ [{}^0 \text{Child\_of}_{wt} {}^0 \text{John}] x ]].$$

Now the literal analysis of the sentence "All John's children are asleep" on its *neutral* reading (that is, without existential presupposition), is best obtained by using the restricted quantifier *All*, because using a general quantifier  $\forall$  would involve implication that does not receive mention in the sentence. Composing the quantifier with the set of John's children at the world/time pair of evaluation, [ ${}^0 \text{All} [{}^0 \text{Child\_of}_{wt} {}^0 \text{John}]$ ], we obtain the set of all supersets of John's children in *w* at *t*. The sentence claims that the population of those who are asleep,  ${}^0 \text{Sleep}_{wt}$ , is one such superset:

$$\lambda w \lambda t [ [{}^0 \text{All} [{}^0 \text{Child\_of}_{wt} {}^0 \text{John}] ] {}^0 \text{Sleep}_{wt} ].$$

The schematic analysis of sentence (3) on its topic-like reading that comes with the presupposition that John have children translates into this procedure:

<sup>27</sup> See (Strawson, 1952, in particular pp. 173ff.)

(3<sup>s</sup>)  $\lambda\omega\lambda t$  [If [<sup>0</sup> $\exists\lambda x$  [[<sup>0</sup>Child\_of<sub>wt</sub><sup>0</sup>John]  $x$ ]] then [[<sup>0</sup>All [<sup>0</sup>Child\_of<sub>wt</sub><sup>0</sup>John]] <sup>0</sup>Sleep<sub>wt</sub>] else Fail.

To finish the analysis, we must define the *if-then-else* function. This I am going to do in the next paragraph.

### 5.1 The *if-then-else* function

In a programming language the *if-then-else* conditional forces a program to perform different actions depending on whether the specified *condition* evaluates true or else false. This is always achieved by *selectively* altering the control flow based on the specified condition. For this reason, an analysis in terms of material implication,  $\supset$ , or even ‘exclusive or’ as known from propositional logic, is not adequate. The reason is this. Since propositional logic is strictly compositional, *both* the ‘then clause’ *and* the ‘else clause’ are always evaluated. For instance, it might seem that the instruction expressed by “The only number  $n$  such that if  $5 = 5$  then  $n$  equals 1, else  $n$  equals the result of 1 divided by 0” would receive the analysis

$$[{}^0\Gamma \lambda n [[[]^{05=05} \supset [n=^01]] \wedge [{}^- [{}^{05=05} \supset [n=[{}^0Div \ ^01 \ ^00]]]]]]$$

Types:  $\Gamma / (\tau(\sigma\tau))$ ;  $n \rightarrow_v \tau$ ; 0, 1, 5/ $\tau$ ; *Div*/( $\tau\tau$ ): the division function.

But the output of the above procedure should be the number 1 because the *else* clause is never executed. However, due to the strict principle of compositionality that TIL observes, the above analysis fails to produce anything, the construction being improper. For, the Composition [<sup>0</sup>*Div* <sup>0</sup>1 <sup>0</sup>0] does not produce anything: it is improper because the division function takes no value at the argument  $\langle 1, 0 \rangle$ . Thus  $[n = [{}^0Div \ ^01 \ ^00]]$  is *v*-improper for any valuation  $v$ , because the identity relation = does not receive a second argument, and so any other Composition containing the improper Composition [<sup>0</sup>*Div* <sup>0</sup>1 <sup>0</sup>0] as a constituent also comes out *v*-improper. The underlying principle is that partiality is being strictly propagated up. This is the reason why the *if-then-else* connective is often said to denote a *non-strict* function not complying with the principle of compositionality. However, as I wish to argue, there is no cogent reason to settle for non-compositionality. I suggest applying a mechanism known in computer science as *lazy evaluation*. As we have seen, the *procedural* semantics of TIL operates smoothly even at the hyperintensional level of constructions. Thus it enables us to specify a definition of *if-then-else* that meets the compositionality constraint. The analysis of

“If  $P$  then  $C$ , else  $D$ ”

reveals a procedure that decomposes into two phases. First, on the basis of the condition  $P$ , select one of  $C$ ,  $D$  as the procedure to be executed. Second, execute the selected procedure. The first phase, *viz.* selection, is realized by the Composition

$$[{}^0\Gamma^* \lambda c [[P \supset [c = {}^0C]] \wedge [{}^- P \supset [c = {}^0D]]]]$$

Types:  $P \rightarrow_v o$  (the condition of the choice between the execution of  $C$  or of  $D$ );  $C, D / *_{ni}$ ; variable  $c \rightarrow_v *_{ni}$ ;  $\Gamma^* / (*_{ni}(o *_{ni}))$ : the singularizer.

The Composition  $[[P \supset [c = {}^0C]] \wedge [{}^- P \supset [c = {}^0D]]]$  *v*-constructs **T** in two cases. If  $P$  *v*-constructs **T** then the variable  $c$  receives as its value the *construction*  $C$ , and if  $P$  *v*-constructs **F** then the variable  $c$  receives the *construction*  $D$  as its value. In either case the set *v*-constructed by

$\lambda c [[P \supset [c=^0C]] \wedge [\neg P \supset [c=^0D]]]$  is a singleton whose element is a construction. Applying  $I^*$  to this set returns as its value the only member of the set, i.e. either  $C$  or  $D$ .<sup>28</sup>

Second, the chosen construction  $c$  is executed. To execute it we apply Double Execution; see Def. 2, vi). As a result, the schematic analysis of “If  $P$  then  $C$ , else  $D$ ” turns out to be

$$(*) \quad {}^2[0I^* \lambda c [[P \supset [c=^0C]] \wedge [\neg P \supset [c=^0D]]]]$$

Note that the evaluation of the first phase does not involve the execution of either of  $C$  or  $D$ . In this phase these constructions figure only as arguments of other functions. In other words, we operate at hyperintensional level. The second phase of execution turns the level down to intensional or extensional one. Thus we define:

**Definition 4** (*If-then-else, if-then-else-fail*). Let  $p/*_n \rightarrow_v o$ ;  $c, d_1, d_2/*_{n+1} \rightarrow *_n$ ;  ${}^2c, {}^2d_1, {}^2d_2 \rightarrow_v \alpha$ . Then the polymorphic functions *if-then-else* and *if-then-else-fail* of types  $(\alpha o*_n*_n)$ ,  $(\alpha o*_n)$ , respectively, are defined as follows:

$${}^0\text{if-then-else} = \lambda p d_1 d_2 {}^2[0I^* \lambda c [[p \supset [c = d_1]] \wedge [\neg p \supset [c = d_2]]]]$$

$${}^0\text{if-then-else-fail} = \lambda p d_1 {}^2[0I^* \lambda c [[p \supset [c = d_1]] \wedge [\neg p \supset {}^0F]]]$$

Now we are ready to specify a **general analytic schema of an (empirical) sentence  $S$  associated with a presupposition  $P$** . In a technical jargon of English the evaluation instruction can be formulated as follows:

At any  $\langle w, t \rangle$ -pair do this:

if  $P_{wt}$  is true then evaluate  $S_{wt}$ , else *Fail* (to produce a truth-value).

Let  $P/*_n \rightarrow o_{\tau_0}$  be a construction of a presupposition,  $S/*_n \rightarrow o_{\tau_0}$  the meaning of the sentence  $S$  and  $c/*_{n+1} \rightarrow_v *_n$  a variable. Then the corresponding TIL construction is this:

$$\lambda w \lambda t [{}^0\text{if-then-else-fail } P_{wt} {}^0S_{wt}] =$$

$$\lambda w \lambda t {}^2[0I^* \lambda c [[P_{wt} \supset [c = {}^0S_{wt}]] \wedge [\neg P_{wt} \supset {}^0F]]]$$

The evaluation of  $S$  for any  $\langle w, t \rangle$ -pair depends on whether the presupposition  $P$  is true at  $\langle w, t \rangle$ . If true, the singleton  $v$ -constructed by  $\lambda c [ \dots ]$  contains as the only construction to be executed  ${}^0S_{wt}$  that is afterwards double executed. The first execution produces  $S_{wt}$  and the second execution produces a truth-value. If  $\neg P_{wt}$   $v$ -constructs  $\mathbf{T}$ , then the second conjunct becomes the Composition  $[{}^0\mathbf{T} \supset {}^0\mathbf{F}]$  and thus we get  $\lambda c {}^0\mathbf{F}$ . The  $v$ -constructed set is *empty*. Hence,  $[I^* \lambda c {}^0\mathbf{F}]$  is  $v$ -improper, and the Double Execution *fails* to produce a truth-value.

Now we can finish the analysis of Strason’s example (3). *First*, make a choice between executing the Composition  $[[{}^0\text{All } [{}^0\text{Child\_of\_wt } {}^0\text{John}] {}^0\text{Sleep\_wt}]$  and a  $v$ -improper construction that fails to produce a truth-value. If the Composition  $[{}^0\exists \lambda x [[{}^0\text{Child\_of\_wt } {}^0\text{John}] x]]$   $v$ -constructs  $\mathbf{T}$  then the former, else the latter. The choice itself is realized by this Composition:

$$[{}^0I^* \lambda c [[\exists x [[{}^0\text{Child\_of\_wt } {}^0\text{John}] x] \supset [c = {}^0[{}^0\text{All } [{}^0\text{Child\_of\_wt } {}^0\text{John}] {}^0\text{Sleep\_wt}]]] \wedge [\neg \exists x [[{}^0\text{Child\_of\_wt } {}^0\text{John}] x] \supset {}^0\mathbf{F}]]]$$

<sup>28</sup> In case  $P$  is  $v$ -improper the singleton is empty and *no* construction is selected to be executed so the execution aborts.

Second, execute the chosen construction. To this end we apply *Double Execution*:

$${}^2[{}^0I^*\lambda c [[\exists x [[{}^0Child\_of_{wt}{}^0John] x] \supset [c = {}^0[[{}^0All [{}^0Child\_of_{wt}{}^0John]] {}^0Sleep_{wt}]]] \\ \wedge [\neg \exists x [[{}^0Child\_of_{wt}{}^0John] x] \supset {}^0F]]]$$

The evaluation of this construction for any  $\langle w, t \rangle$  depends on whether the presupposition condition  $\exists x [[{}^0Child\_of_{wt}{}^0John] x]$  is true at  $\langle w, t \rangle$ :

a.  $\exists x [[{}^0Child\_of_{wt}{}^0John] x] \rightarrow_v \mathbf{T}$ .

Then  $\lambda c [{}^0T \supset [c = {}^0[[{}^0All [{}^0Child\_of_{wt}{}^0John]] {}^0Sleep_{wt}] \wedge [{}^0F \supset {}^0F]]]$   $v$ -constructs this singleton:  $\{[{}^0[[{}^0All [{}^0Child\_of_{wt}{}^0John]] {}^0Sleep_{wt}]]\}$ . Hence the value of  $I^*$  is its only member and we have:

$${}^2[{}^0I^*\lambda c [{}^0T \supset [c = {}^0[[{}^0All [{}^0Child\_of_{wt}{}^0John]] {}^0Sleep_{wt}] \wedge [{}^0F \supset {}^0F]]] = \\ {}^{20}[[{}^0All [{}^0Child\_of_{wt}{}^0John]] {}^0Sleep_{wt}] = [[{}^0All [{}^0Child\_of_{wt}{}^0John]] {}^0Sleep_{wt}].$$

b.  $\exists x [[{}^0Child\_of_{wt}{}^0John] x] \rightarrow_v \mathbf{F}$ .

Then  $\lambda c [{}^0F \supset [c = {}^0[[{}^0All [{}^0Child\_of_{wt}{}^0John]] {}^0Sleep_{wt}] \wedge [{}^0T \supset {}^0F]]] = \lambda c {}^0F$ . The  $v$ -constructed set is *empty*, function  $I^*$  being undefined at such set. Hence,  ${}^2[{}^0I^*\lambda c {}^0F]$  is  $v$ -improper, *fails*.

Finally, we must abstract over the values of  $w$  and  $t$  in order to construct a proposition of type  $o_{\tau_w}$  denoted by the sentence. The resulting analysis of (3) is this:

$$(3^*) \quad \lambda w \lambda t {}^2[{}^0I^*\lambda c [\exists x [[{}^0Child\_of_{wt}{}^0John] x] \supset [c = {}^0[[{}^0All [{}^0Child\_of_{wt}{}^0John]] {}^0Sleep_{wt}]] \\ \wedge [\neg \exists x [[{}^0Child\_of_{wt}{}^0John] x] \supset {}^0F]]]$$

In the interest of better readability I will in the remainder use a more standard notation. Hence instead of either “ $\lambda w \lambda t [{}^0If\text{-then-else-fail } P_{wt} {}^0S_{wt}]$ ” or “ $\lambda w \lambda t {}^2[{}^0I^*\lambda c [[P_{wt} \supset [c = {}^0S_{wt}]] \wedge [\neg P_{wt} \supset {}^0F]]]$ ” I will simply write “ $\lambda w \lambda t [If P_{wt} \text{ then } S_{wt} \text{ else Fail}]$ ”.

## 5.2 Additional examples

Consider now another pair of sentences differing only in terms of topic-focus articulation:

(4) “The *global financial and economic crisis* was caused by the Bank of America.”

(5) “The *Bank of America* caused the global financial and economic crisis.”

While (4) not only entails but also presupposes that there be a global financial and economic crisis, the truth-conditions of (5) are different, as our analysis clarifies. First, (4) as well as

(4’) “The *global financial and economic crisis* was not caused by Bank of America”

are about the global crisis, and that there is such a crisis is not only entailed but also presupposed by both sentences. The instruction encoded by (4) formulated in logician’s English is this:

“If there is a global crisis *then* return **T** or **F** according as the crisis was caused by the Bank of America, *else fail* (to produce a truth-value)”

Since every TIL analysis is fully compositional, we first need to analyse the particular constituents of this instruction, and then combine these constituents into the construction expressed by the sentence. As always, we start with assigning types to the objects that receive mention in the sentence. Simplifying a bit, let the objects be: *Crisis*/ $o_{\tau_w}$ : the

proposition that there is a global financial and economic crisis;  $Cause/(o_{10}o_{\tau_{10}})_{\tau_{10}}$ : the relation-in-intension between an individual and a proposition which has been caused to be true by the individual;  $Bank\_of\_America/\iota_{\tau_{10}}$ : the individual office occupiable by a corporation belonging to the American financial institutions.

A schematic analysis of (4) comes down to this procedure:

$$\lambda\omega\lambda t \text{ [If } {}^0Crisis_{wt} \text{ then } [{}^0True_{wt} \lambda\omega\lambda t [{}^0Cause_{wt} {}^0Bank\_of\_America_{wt} {}^0Crisis]] \text{ else Fail}]$$

Here we are again using the propositional property *True* in the then-clause, because this clause occurs in the focus of the sentence, and thus with *de dicto* supposition. The existence of the Bank of America is not presupposed.

The truth-conditions of the other reading with ‘Bank of America’ as topic are different. Now the sentence (5) is about the Bank of America (topic), ascribing to this corporation the property that it caused the crisis (focus). Thus the scenario of truly asserting that (5) is *not true* can be, for instance, this. Though it is true that the Bank of America played a major role in risky investments in China, the President of USA played a positive role in enhancing financial-market transparency and passed new laws that prevented a global crisis from arising. Or, a less optimistic scenario is thinkable. The global financial and economic crisis is not due to the Bank of America’s bad investments but because in the era of globalisation the market economy is unpredictable, hence uncontrollable. Hence, that there is a crisis is not presupposed by (5), and its analysis is this Closure:

$$\lambda\omega\lambda t \text{ [If } [{}^0Exist_{wt} {}^0Bank\_of\_America] \text{ then } [{}^0True_{wt} \lambda\omega\lambda t [{}^0Cause_{wt} {}^0Bank\_of\_America_{wt} {}^0Crisis]] \text{ else Fail}]$$

Note that (5) presupposes the existence of the Bank of America, while the existence of the crisis is not presupposed. Yet, if (5) is true, then the existence of the crisis can be validly inferred. To capture such truth-conditions, we need to refine the analysis. A plausible explication of this phenomenon is this:  $x$  is a cause of a proposition  $p$  iff  $p$  is true and if it is so then  $x$  affected  $p$  so as to become true. Schematically,

$$\lambda\omega\lambda t [{}^0Cause_{wt} x p] = \lambda\omega\lambda t [p_{wt} \wedge [p_{wt} \supset [{}^0Affect_{wt} x p]]]$$

Types:  $Cause, Affect/(o\alpha o_{\tau_{10}})_{\tau_{10}}; x \rightarrow \alpha, \alpha$ : any type;  $p \rightarrow o_{\tau_{10}}$ .

If  $x$  is not a cause of  $p$ , then either  $p$  is not true or  $p$  is true but  $x$  did not affect  $p$  so as to become true:  $\lambda\omega\lambda t \neg[{}^0Cause_{wt} x p] = \lambda\omega\lambda t [\neg p_{wt} \vee [p_{wt} \wedge \neg[{}^0Affect_{wt} x p]]]$ .<sup>29</sup> By applying such an explication to our sentence, the construction corresponding to the ‘then clause’, viz.  $\lambda\omega\lambda t [{}^0Cause_{wt} {}^0Bank\_of\_America_{wt} {}^0Crisis]$ , is refined to:

$$\lambda\omega\lambda t [{}^0Crisis_{wt} \wedge [{}^0Crisis_{wt} \supset [{}^0Affect_{wt} {}^0Bank\_of\_America_{wt} {}^0Crisis]]]$$

This Closure entails that there is a crisis, which is the desired (logical, though not economic) outcome.

The topic-focus ambiguity also crops up in the case of propositional and notional attitudes, as noted in the Introduction.<sup>30</sup> Imagine one is referring to the tragedy in Dallas, November

<sup>29</sup> For the sake of simplicity, I ignore here the past tense ‘affected’; a more precise analysis is this:  $\lambda\omega\lambda t [p_{wt} \wedge [p_{wt} \supset \exists t' [[t' < t] \wedge [{}^0Affect_{wt} x p]]]]$ .

<sup>30</sup> For an analysis of propositional attitudes *de dicto* and *de re*, see (Duží *et al.*, 2010a, § 5.1.2).

22, 1963, by “The police were seeking the murderer of JFK, but never found him”. The sentence is again ambiguous due to a difference in topic-focus articulation, as evidenced by (6) and (7):

- (6) The *police* were seeking the murderer of JFK, but never found him.  
 (7) The police were seeking *the murderer of JFK*, but never found him.

The existence of the murderer of JFK is not presupposed by (6), unlike (7). The sentence (6) can be true in such states-of-affairs where JFK was not murdered, unlike (7). The latter can be reformulated in a less ambiguous way as “*The murderer of JFK* was looked for by the police, but was never found”. This sentence expresses the construction

$$\lambda\omega\lambda t \text{ [If } [{}^0\text{Exist}_{wt} \lambda\omega\lambda t [{}^0\text{Murderer\_of}_{wt} {}^0\text{JFK}]] \text{ then } [{}^0\text{Seek}_{wt} {}^0\text{Police } \lambda\omega\lambda t [{}^0\text{Murderer\_of}_{wt} {}^0\text{JFK}]] \wedge \neg[{}^0\text{Find}_{wt} {}^0\text{Police } \lambda\omega\lambda t [{}^0\text{Murderer\_of}_{wt} {}^0\text{JFK}]] \text{ else Fail.}]$$

Types: *Seek*, *Find*/ $(o\iota\iota_{\tau\omega})_{\tau\omega}$ : the relation-in-intension between an individual and an individual office (the seeker wants to find out who is the holder of the office); *Police*/ $\iota$ ; *Murderer\_of*/ $(\iota\iota)_{\tau\omega}$ ; *JFK*/ $\iota$ .<sup>31</sup>

On the other hand, the analysis of (6) comes down to this construction:

$$\lambda\omega\lambda t \text{ [[} [{}^0\text{Seek}_{wt} {}^0\text{Police } [\lambda\omega\lambda t [{}^0\text{Murderer\_of}_{wt} {}^0\text{JFK}]]] \wedge \neg[{}^0\text{Find}_{wt} {}^0\text{Police } [\lambda\omega\lambda t [{}^0\text{Murderer\_of}_{wt} {}^0\text{JFK}]]]].$$

If the police did not find the murderer then either the murderer did not exist or the murderer did exist; only the search was not successful. However, if the foregoing search was successful, then it is true that police found the murderer and the murderer exists. Hence, a successful search, i.e. *finding* after a foregoing search, merely entails that the murderer exists and the following argument is valid:

$$\lambda\omega\lambda t [{}^0\text{Find}_{wt} {}^0\text{Police } [\lambda\omega\lambda t [{}^0\text{Murderer\_of}_{wt} {}^0\text{JFK}]]] \\ \hline \lambda\omega\lambda t [{}^0\text{Exist}_{wt} [\lambda\omega\lambda t [{}^0\text{Murderer\_of}_{wt} {}^0\text{JFK}]]]$$

In order to logically reproduce this entailment, we explicate *finding after a foregoing search* in a manner similar to *causing* ( $x \rightarrow_v \iota$ ;  $c \rightarrow_v \iota_{\tau\omega}$ ; *Success\_Search*/ $(o\iota\iota_{\tau\omega})_{\tau\omega}$ ):

$$\lambda\omega\lambda t [{}^0\text{Find}_{wt} x c] = \lambda\omega\lambda t \text{ [[} [{}^0\text{Exist}_{wt} c] \wedge [[{}^0\text{Exist}_{wt} c] \supset [{}^0\text{Success\_Search}_{wt} x c]]]; \\ \lambda\omega\lambda t \neg[{}^0\text{Find}_{wt} x c] = \lambda\omega\lambda t \text{ [}\neg[{}^0\text{Exist}_{wt} c] \vee [[{}^0\text{Exist}_{wt} c] \wedge \neg[{}^0\text{Success\_Search}_{wt} x c]]].$$

Thus the analysis of such an explication of the sentence “The *police* found the murderer of JFK” is this Closure:

$$\lambda\omega\lambda t \text{ [[} [{}^0\text{Exist}_{wt} \lambda\omega\lambda t [{}^0\text{Murderer\_of}_{wt} {}^0\text{JFK}]] \wedge [[{}^0\text{Exist}_{wt} \lambda\omega\lambda t [{}^0\text{Murderer\_of}_{wt} {}^0\text{JFK}]] \supset [{}^0\text{Success\_Search}_{wt} {}^0\text{Police } \lambda\omega\lambda t [{}^0\text{Murderer\_of}_{wt} {}^0\text{JFK}]]]]$$

From this analysis one can validly infer that the murderer exists and that the search was successful, just as we ought to be able to. And if the so constructed proposition is not true,

<sup>31</sup> For the sake of simplicity, past tense and anaphoric reference are ignored. For a more detailed analysis of this kind of seeking and finding, see, for instance, (Duží 2003) or (Duží *et al.*, 2010a, § 5.2.2).

then the murderer does not exist or the murder does exist, only the search did not meet with success.

The next example I am going to analyse is again due to (Hajičová, 2008):

(8) “John only introduced *Bill* to Sue.”

(9) “John only introduced *Bill* to *Sue*.”

Leaving aside the possible disambiguation “John introduced only *Bill* to Sue” vs. “John introduced *Bill* only to *Sue*”, (8) can be truly affirmed only in a situation where John did not introduce other people to Sue than *Bill*. This is not the case of (9). This sentence can be true in a situation where John introduced other people to Sue, but the only person *Bill* was introduced to by John was Sue. Hence the presuppositions of (8) and (9) are constructed by these Closures:

Presupposition of (8):  $\lambda\omega\lambda t [\forall x [[{}^0Int\_to_{wt}{}^0John\ x\ {}^0Sue] \supset [x = {}^0Bill]]]$

Presupposition of (9):  $\lambda\omega\lambda t [\forall y [[{}^0Int\_to_{wt}{}^0John\ {}^0Bill\ y] \supset [y = {}^0Sue]]]$

The construction *C* that is to be executed in case a relevant presupposition is true is here the Closure  $\lambda\omega\lambda t [{}^0Int\_to_{wt}{}^0John\ {}^0Bill\ {}^0Sue]$ . Types:  $Int\_to/(ouu)_{\tau\omega}$ : a relation-in-intension between the individual *who* does the introducing, another individual *who* is introduced, and yet another individual to *whom* the second individual was introduced; *John, Sue, Bill*/ι.

The resulting analyses are

(8\*)  $\lambda\omega\lambda t [If\ \forall x [[{}^0Int\_to_{wt}{}^0John\ x\ {}^0Sue] \supset [x = {}^0Bill]]\ then\ [{}^0Int\_to_{wt}{}^0John\ {}^0Bill\ {}^0Sue]$   
else fail];

(9\*)  $\lambda\omega\lambda t [If\ \forall y [[{}^0Int\_to_{wt}{}^0John\ {}^0Bill\ y] \supset [y = {}^0Sue]]\ then\ [{}^0Int\_to_{wt}{}^0John\ {}^0Bill\ {}^0Sue]$   
else fail].

Using technical jargon, the truth-conditions constructed by the construction (8\*) are, “If the only person that was introduced by John to Sue is *Bill*, then it is true that John introduced only *Bill* to Sue, otherwise there is no truth-value”. Similarly for (9\*).

For the last example, consider the sentence

“All students of VSB-TU Ostrava who signed up for the Logic course  
in the winter term of 2011 passed the final exam.”

There are again two readings matching two possible scenarios.

*Scenario 1:* We are talking about the students of VSB-Technical University Ostrava, and somebody then asks, “What about the students of VSB-TU Ostrava who signed up for the Logic course in the winter term of 2011 – how did they do?”. The answer is, “They did well, they all passed the final exam”.

In this case the topic of the sentence is the students enrolled in the Logic course. Thus the sentence comes with the presupposition that there should be students of VSB-TU Ostrava having signed up for Logic in the winter term of 2011. If this presupposition is not satisfied (for instance, because the course runs only in the summer term) then the sentence is neither true nor false, leaving a truth-value gap. For the negated sentence cannot be true, either: “Some students of VSB-TU Ostrava who signed up for Logic in the winter term of 2011 did



not pass the final exam". Moreover, the positive sentence merely entails (and so does not presuppose) that the final exam has taken place. This is so because the sentence can be false for either of two reasons: Either some of the students did not succeed, or none of the students succeeded because the exam has yet to take place.

*Scenario 2:* The topic is the final exam. Somebody asks, "What about the final exam in Logic, what are the results?" One possible answer is, "All students passed". Now the sentence presupposes that the final exam have already taken place. If it has not then the sentence is neither true nor false, because the negated sentence ("The final exam has not been passed by all students ...") cannot be true, either. In this situation the (positive) sentence does not presuppose, but only entails, that some students signed up for the course.

The logical machinery of TIL, thanks not least to the application of Definition 4, makes it easy to properly distinguish between those two non-equivalent readings. In the situation corresponding to the first scenario the meaning of the sentence is this Closure:

$$\lambda w \lambda t \text{ [If } [\exists [\text{}^0\text{Students\_enrolled\_in}_{wt} \text{}^0\text{Logic}]] \text{ then } [[\text{}^0\text{All } [\text{}^0\text{Students\_enrolled\_in}_{wt} \text{}^0\text{Logic}]] \text{ } [\text{}^0\text{Passed}_{wt} \text{}^0\text{Exam}]] \text{ else Fail}]}$$

The second scenario receives this Closure as analysis:

$$\lambda w \lambda t \text{ [If } \text{Exam}_{wt} \text{ than } [[\text{}^0\text{All } [\text{}^0\text{Students\_enrolled\_in}_{wt} \text{}^0\text{Logic}]] \text{ } [\text{}^0\text{Passed}_{wt} \text{}^0\text{Exam}]] \text{ else Fail}]}$$

*Types:*  $\exists/(o(o))$ : the existential quantifier; *Students\_enrolled\_in*/ $((o)t)_{\tau o}$ : an attribute (i.e. empirical function) that dependently on a given state-of-affairs assigns to an individual a set of individuals; *Logic*/ $t$  (for the sake of simplicity); *All*/ $((o(o))(o))$ : a restricted quantifier, which is a function assigning to a set  $S$  of individuals the set of all supersets of  $S$ ; *Passed*/ $((o)_{\tau o})_{\tau o}$ : a function that dependently on a given state-of-affairs associates a proposition (in this case an event) with the set of individuals (who are the successful actors of the event); *Exam*/ $o_{\tau o}$ : the proposition that the final exam takes place.<sup>32</sup>

## 6. Conclusion

In this chapter I brought out the *semantic*, as opposed to pragmatic, character of the ambivalence stemming from topic-focus articulation. The procedural semantics of TIL provided rigorous analyses such that sentences differing only in their topic-focus articulation were assigned different constructions producing different propositions (truth-conditions) and having different consequences. I showed that a definite description occurring in the topic of a sentence with *de re* supposition corresponds to the Strawsonian analysis of definite descriptions, while a definite description occurring in the focus with *de dicto* supposition corresponds to the Russellian analysis. While the clause standing in topic

<sup>32</sup> For the sake of simplicity we are ignoring the past tense of the sentence. For the TIL analysis of tenses, see (Duží et al., 2010a, § 2.5.2). Similarly as above, see the sentence (3), we again apply the restricted quantifier *All* in the analysis of the clause "All students who signed up for Logic passed the exam".

position triggers a presupposition, a focus clause usually entails rather than presupposes another proposition. Thus both opponents and proponents of Russell's quantificational analysis of definite descriptions are partly right and partly wrong.

Moreover, the proposed analysis of the Russellian reading does not deprive definite descriptions of their meaning. Just the opposite; 'the *F*' receives a context-invariant meaning. What is dependent on context is the way this (one and the same) meaning is used. Thus I also demonstrated that Donnellan-style referential and attributive uses of an occurrence of 'the *F*' do not bring about a shift of meaning of 'the *F*'. Instead, one and the same context-invariant meaning is a constituent of different procedures that behave in different ways.

The proposed analysis of topic-focus ambivalence was then generalized to sentences containing not only singular clauses like 'the *F*' but also general clauses like 'John's children', 'all students' in the topic or focus of a sentence. As a result, I proposed a general analytic schema for sentences equipped with a presupposition. This analysis makes use of the definition of the *if-then-else* function that complies with the desirable principle of compositionality. This is also my novel contribution to the old problem of the semantic character of the specification of the *if-then-else* function. I demonstrated the method by analysing several examples including notional attitudes like seeking and finding.

The moral to be drawn from my contribution is this. Logical analysis disambiguates ambiguous expressions, but cannot dictate *which* disambiguation is the intended one (leaving room for pragmatics here). Yet, our fine-grained method of analysis contributes to language disambiguation by making its hidden features *explicit* and *logically tractable*. In case there are more senses of a sentence we furnish the sentence with different TIL logical forms. Having a formal, fine-grained encoding of linguistic senses at our disposal, we are in a position to automatically infer the relevant consequences.

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