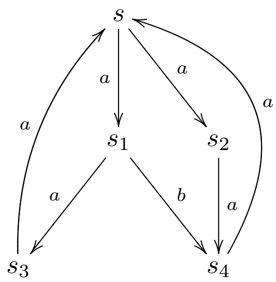
Tutorial 5 - Solutions

Exercise 1*

Consider the following labelled transition system.



1. Decide whether the state s satisfies the following formulae of Hennessy-Milner logic:
   - $s \models \langle a \rangle tt$
   - $s \not\models \langle b \rangle tt$
   - $s \not\models [a] ff$
   - $s \models [b] ff$
   - $s \not\models [a]\langle b \rangle tt$
   - $s \models \langle a \rangle\langle b \rangle tt$
   - $s \models [a]\langle a \rangle[a][b] ff$
   - $s \models \langle a \rangle\big(\langle a \rangle tt \wedge \langle b \rangle tt\big)$
   - $s \models [a]\big(\langle a \rangle tt \vee \langle b \rangle tt\big)$
   - $s \not\models \langle a \rangle\big([b][a] ff \wedge \langle b \rangle tt\big)$
   - $s \not\models \langle a \rangle\big([a](\langle a \rangle tt \wedge [b] ff) \wedge \langle b \rangle ff\big)$
2. Compute the following sets according to the denotational semantics for Hennessy-Milner logic.
   -
$$\begin{aligned}
[\![[a][b] ff]\!] &= [\cdot a \cdot][\![[b] ff]\!] \\
&= [\cdot a \cdot][\cdot b \cdot][\![ff]\!] \\
&= [\cdot a \cdot][\cdot b \cdot]\emptyset \\
&= [\cdot a \cdot]\{P \,|\, \forall P'. P \xrightarrow{b} P' \Rightarrow P' \in \emptyset\} \\
&= [\cdot a \cdot]\{s, s_3, s_2, s_4\} \\
&= \big\{P \,|\, \forall P'. P \xrightarrow{a} P' \Rightarrow P' \in \{s, s_3, s_2, s_4\}\big\} \\
&= \{s_1, s_2, s_3, s_4\}
\end{aligned}$$
   -
$$\begin{aligned}
[\![\langle a \rangle\big(\langle a \rangle tt \wedge \langle b \rangle tt\big)]\!] &= \langle \cdot a \cdot \rangle [\![\langle a \rangle tt \wedge \langle b \rangle tt]\!] \\
&= \langle \cdot a \cdot \rangle \big([\![\langle a \rangle tt]\!] \cap [\![\langle b \rangle tt]\!]\big) \\
&= \langle \cdot a \cdot \rangle \big(\langle \cdot a \cdot \rangle Proc \cap \langle \cdot b \cdot \rangle Proc\big) \\
&= \langle \cdot a \cdot \rangle \big(\{s, s_1, s_2, s_3, s_4\} \cap \{s_1\}\big) \\
&= \langle \cdot a \cdot \rangle \{s_1\} \\
&= \{s\}
\end{aligned}$$

- $$\begin{aligned}[\![[a][a][b]ff]\!] &= [\cdot a\cdot][\cdot a\cdot][\cdot b\cdot]\emptyset \\ &= [\cdot a\cdot][\cdot a\cdot]\{s, s_2, s_3, s_4\} \\ &= [\cdot a\cdot]\{s_1, s_2, s_3, s_4\} \\ &= \{s, s_1, s_2\}\end{aligned}$$

- $$\begin{aligned}[\![[a]\big(\langle a\rangle tt \vee \langle b\rangle tt\big)]\!] &= [\cdot a\cdot][\![\langle a\rangle tt \vee \langle b\rangle tt]\!] \\ &= [\cdot a\cdot]\big(\langle\cdot a\cdot\rangle Proc \cup \langle\cdot b\cdot\rangle Proc\big) \\ &= [\cdot a\cdot]\{s, s_1, s_2, s_3, s_4\} \\ &= \{s, s_1, s_2, s_3, s_4\}\end{aligned}$$

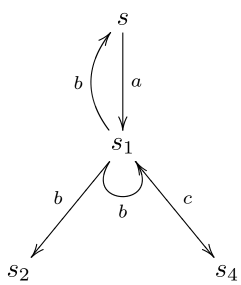
Exercise 2

Find (one) labelled transition system with an initial state s such that it satisfies (at the same time) the following properties:

- $s \models \langle a\rangle(\langle b\rangle\langle c\rangle tt \wedge \langle c\rangle tt)$
- $s \models \langle a\rangle\langle b\rangle([a]ff \wedge [b]ff \wedge [c]ff)$
- $s \models [a]\langle b\rangle([c]ff \wedge \langle a\rangle tt)$

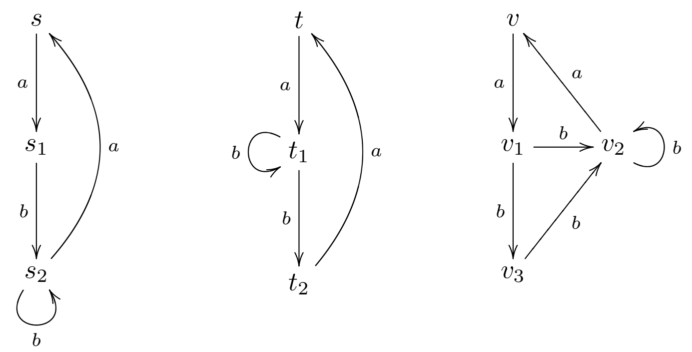
Solution

One possible solution is as follows.



Exercise 3*

Consider the following labelled transition system.



It it true that $s \not\sim t$, $s \not\sim v$ and $t \not\sim v$. Find a distinguishing formula of Hennessy-Milner logic for the pairs

- s and t
- s and v
- t and v.

Solution

Distingushing HML-formulae are as follows.

- Let $F_1 = \langle a \rangle [b] \langle b \rangle tt$. Then $s \models F_1$, but $t \not\models F_1$.
- Let $F_2 = \langle a \rangle [b] \langle a \rangle tt$. Then $s \models F_2$ but $v \not\models F_3$.
- Let $F_3 = \langle a \rangle \langle b \rangle \big(\langle a \rangle tt \wedge \langle b \rangle tt \big)$. Then $t \not\models F_3$ but $v \models F_3$.

Exercise 4*

For each of the following CCS expressions decide whether they are strongly bisimilar and if not, find a distinguishing formula in Hennessy-Milner logic.

- $b.a.Nil + b.Nil$ and $b.(a.Nil + b.Nil)$
  - They are not bisimilar. Let $F_1 = [b] \langle b \rangle tt$. Then $b.a.Nil + b.Nil \not\models F_1$ but $b.(a.Nil + b.Nil) \models F_1$.
- $a.(b.c.Nil + b.d.Nil)$ and $a.b.c.Nil + a.b.d.Nil$
  - They are not bisimilar. Let $F_2 = [a] \big(\langle b \rangle \langle c \rangle tt \wedge \langle b \rangle \langle d \rangle tt \big)$. Then $a.(b.c.Nil + b.d.Nil) \models F_2$ but $a.b.c.Nil + a.b.d.Nil \not\models F_2$.
- $a.Nil \,|\, b.Nil$ and $a.b.Nil + b.a.Nil$
  - They are bisimilar.
- $(a.Nil \,|\, b.Nil) + c.a.Nil$ and $a.Nil \,|\, (b.Nil + c.Nil)$
  - They are not bisimilar. Let $F_3 = [a] \langle c \rangle tt$. Then $(a.Nil \,|\, b.Nil) + c.a.Nil \not\models F_3$ but $a.Nil \,|\, (b.Nil + c.Nil) \models F_3$.

Home exercise: verify your claims in CWB (use the `strongeq` and `checkprop` commands) and check whether you found the shortest distinguishing formula (use the `dfstrong` command).