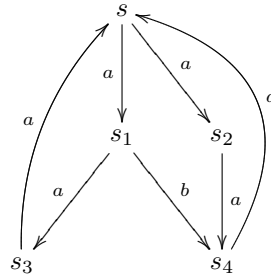# Tutorial 5

Exercise 1*

Consider the following labelled transition system.

1. Decide whether the state s satisfies the following formulae of Hennessy-Milner logic:

   - $s \overset{?}{\models} \langle a \rangle \mathit{tt}$
   - $s \overset{?}{\models} \langle b \rangle \mathit{tt}$
   - $s \overset{?}{\models} [a] \mathit{ff}$
   - $s \overset{?}{\models} [b] \mathit{ff}$
   - $s \overset{?}{\models} [a] \langle b \rangle \mathit{tt}$
   - $s \overset{?}{\models} \langle a \rangle \langle b \rangle \mathit{tt}$
   - $s \overset{?}{\models} [a] \langle a \rangle [a][b] \mathit{ff}$
   - $s \overset{?}{\models} \langle a \rangle (\langle a \rangle \mathit{tt} \wedge \langle b \rangle \mathit{tt})$
   - $s \overset{?}{\models} [a] (\langle a \rangle \mathit{tt} \vee \langle b \rangle \mathit{tt})$
   - $s \overset{?}{\models} \langle a \rangle ([b][a] \mathit{ff} \wedge \langle b \rangle \mathit{tt})$
   - $s \overset{?}{\models} \langle a \rangle ([a](\langle a \rangle \mathit{tt} \wedge [b] \mathit{ff}) \wedge \langle b \rangle \mathit{ff})$

2. Compute the following sets according to the denotational semantics for Hennessy-Milner logic.
   - $[\![[a][b] \mathit{ff}]\!] = ?$
   - $[\![\langle a \rangle (\langle a \rangle \mathit{tt} \wedge \langle b \rangle \mathit{tt})]\!] = ?$
   - $[\![[a][a][b] \mathit{ff}]\!] = ?$
   - $[\![[a] (\langle a \rangle \mathit{tt} \vee \langle b \rangle \mathit{tt})]\!] = ?$
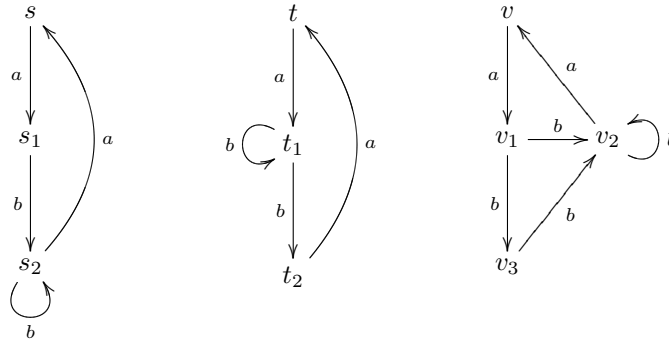
Exercise 2

Find (one) labelled transition system with an initial state s such that it satisfies (at the same time) the following properties:

- $s \models \langle a \rangle (\langle b \rangle \langle c \rangle \mathit{tt} \wedge \langle c \rangle \mathit{tt})$

- $s \models \langle a \rangle \langle b \rangle ([a] \mathit{ff} \wedge [b] \mathit{ff} \wedge [c] \mathit{ff})$

- $s \models [a] \langle b \rangle ([c] \mathit{ff} \wedge \langle a \rangle \mathit{tt})$

Exercise 3*

Consider the following labelled transition system.

It it true that $s \not\sim t$, $s \not\sim v$ and $t \not\sim v$. Find a distinguishing formula of Hennessy-Milner logic for the pairs

- s and t
- s and v
- t and v.

Exercise 4*

For each of the following CCS expressions decide whether they are strongly bisimilar and if no, find a distinguishing formula in Hennessy-Milner logic.

- $b.a.Nil + b.Nil$ and $b.(a.Nil + b.Nil)$
- $a.(b.c.Nil + b.d.Nil)$ and $a.b.c.Nil + a.b.d.Nil$
- $a.Nil \mid b.Nil$ and $a.b.Nil + b.a.Nil$
- $(a.Nil \mid b.Nil) + c.a.Nil$ and $a.Nil \mid (b.Nil + c.Nil)$

Home exercise: verify your claims in CWB (use the `strongeq` and `checkprop` commands) and check whether you found the shortest distinguishing formula (use the `dfstrong` command).

Exercise 5 (optional)

Prove that for every Hennessy-Milner formula F and every state $p \in Proc$:

$$p \models F \quad \text{if and only if} \quad p \in \llbracket F \rrbracket.$$

Hint: use structural induction on the structure of the formula F.