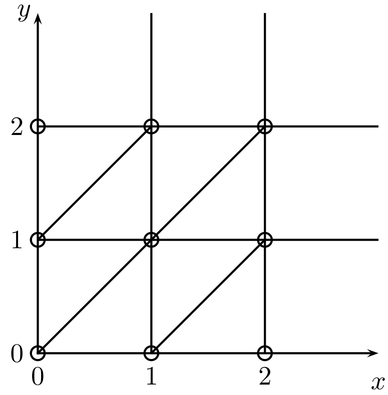
Tutorial 8 - Solutions

Exercise 1

Let $C = \{x, y\}$ be a set of clocks such that $c_x = 2$ and $c_y = 2$.

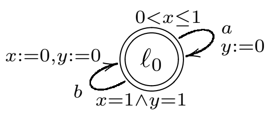
- Draw a picture with all regions for the clocks x and y.



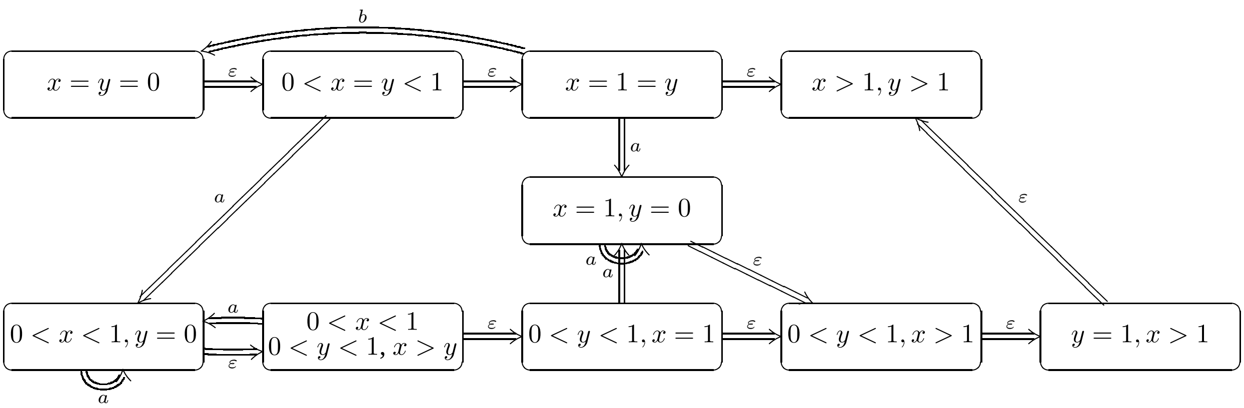
- There are 9 corner points, 22 line segments, and 13 area regions.
- Select four regions (corner point, line, two areas) and describe them by extended clock constraints.
  - Solution (for example): $[x = 0 \wedge y = 0]$, $[0 < x < 1 \wedge 1 < y < 2 \wedge x + 1 = y]$, $[0 < x < 1 \wedge 0 < y < 1 \wedge x < y]$, and $[1 < x < 2 \wedge 0 < y < 1 \wedge x > y + 1]$.
- A formula describing the number of regions for two clocks and maximal constants c_x and c_y: $(c_x + 1)(c_y + 1) + 5c_x c_y + 3(c_x + c_y) + 3$

Exercise 2*

Draw a region graph of the following timed automaton.



All symbolic states contain ℓ_0, so ℓ_0 is omitted in the figure; we also omit the arrows $\overset{\varepsilon}{\Longrightarrow}$ implied by reflexivity and transitivity. Moreover, we construct only the states (regions) which are reachable from the initial state.

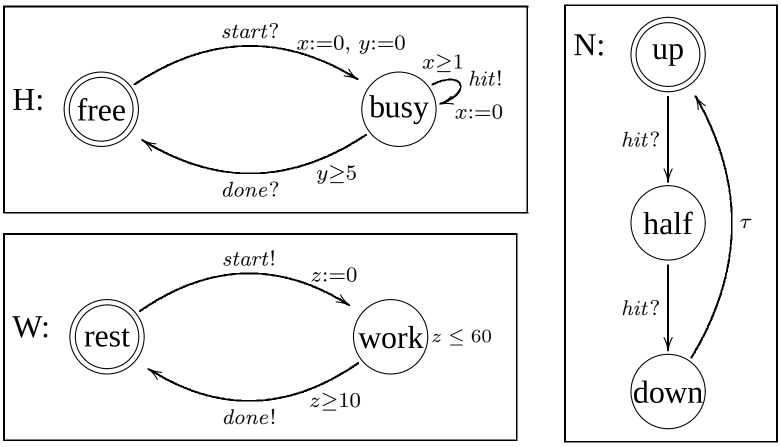


Using the region graph decide whether the following configurations are reachable from the initial configuration.

- (ℓ_0, v) where $v(x) = 0.7$ and $v(y) = 0.61$
  - Solution: Yes, since the symbolic state
$$(\ell_0, [v]) = (\ell_0, 0 < x < 1 \wedge 0 < y < 1 \wedge x > y)$$
is reachable from the initial symbolic state $(\ell_0, x = y = 0)$ of the region graph.
- (ℓ_0, v) where $v(x) = 0.2$ and $v(y) = 0.41$
  - Solution: No, since the symbolic state
$$(\ell_0, [v]) = (\ell_0, 0 < x < 1 \wedge 0 < y < 1 \wedge x < y)$$
is *not* reachable from the initial symbolic state $(\ell_0, x = y = 0)$ of the region graph.

Exercise 3

Consider the following network of timed automata from the lecture.



- Give an example of a timed trace in the network above.
  - A timed trace could be as follows:
$$(20, \tau)(40, \tau)(60, \tau)(60, \tau) \cdots$$
An example of a sequence of states could be:
$\big((\text{free,rest,up}), [x = 0, y = 0, z = 0]\big) \xrightarrow{\tau} \big((\text{busy,work,up}), [x = 0, y = 0, z = 0]\big) \xrightarrow{20} \big((\text{busy,work,up}), [x = 20, y = 20, z = 20]\big) \xrightarrow{\tau} \big((\text{busy,work,half}), [x = 0, y = 20, z = 20]\big) \xrightarrow{40} \big((\text{busy,work,half}), [x = 40, y = 60, z = 60]\big) \xrightarrow{\tau} \big((\text{busy,work,down}), [x = 0, y = 60, z = 60]\big) \xrightarrow{\tau} \big((\text{free,rest,down}), [x = 0, y = 60, z = 60]\big) \xrightarrow{\tau} \big((\text{free,rest,up}), [x = 0, y = 60, z = 60]\big) \cdots$
- Which of the following properties are true?
  - A[] (W.rest $\vee$ $z \leq 100$) : **True**
  - E$\langle\rangle$ (W.rest $\wedge$ H.busy) : **False**
  - A$\langle\rangle$ W.rest : **True**
  - E[] H.busy : **False**
  - W.work $--$ $>$ W.rest : **True**