Tutorial 1

Exercise 1
What are all the possible values that can be stored in the variable \( x \) after the execution of the following parallel program?

\[
x := 10; \ (x := x * 2; \ x := x - 11; \ x := x + 2) \mid x := x - 5
\]

Exercise 2

Let \( R \) be a binary relation on a set \( A \). Let us define the binary relation

\[
E \overset{\text{def}}{=} \{(x, x) \mid x \in A\}.
\]

It is trivially true that \( R \cup E \) is a reflexive relation.

- Argue that \( R \cup E \) is a reflexive closure of \( R \).

Exercise 3

Let \( R \) be a binary relation on a set \( A \). Let us define the binary relation

\[
R^{-1} \overset{\text{def}}{=} \{(y, x) \mid (x, y) \in R\}.
\]

- Argue that \( R \cup R^{-1} \) is a symmetric relation.
- Argue that \( R \cup R^{-1} \) is a symmetric closure of \( R \).

Exercise 4*

Let us consider the following labelled transition system.

- Define the labelled transition system as a triple \( (\text{Proc}, \text{Act}, \{ a \rightarrow \mid a \in \text{Act}\}) \).
- What is the reflexive closure of the binary relation \( a \rightarrow \)? (A drawing is fine.)
- What is the symmetric closure of the binary relation \( a \rightarrow \)? (A drawing is fine.)
- What is the transitive closure of the binary relation \( a \rightarrow \)? (A drawing is fine.)

Example 5

Let us consider the following CCS definition of a coffee machine.

\[
\text{CM} \overset{\text{def}}{=} \text{coin.coin.coffee.CM}
\]

- Give a CCS process which describes a coffee machine that may behave like \( \text{CM} \) but may also steal the money it receives and fail at any time.
Example 6

Assume a given labelled transition system $T = (\text{Proc}, \text{Act}, \{ \rightarrow_a \mid a \in \text{Act} \})$ such that the sets $\text{Proc}$ and $\text{Act}$ are finite.

- Does it imply that $\rightarrow_a$ is also a finite set? Why?
- Draw an example of an LTS with four states and two actions.
- How can your example be described by a sequential fragment of CCS (with Nil, action prefixing, nondeterminism and recursive definitions of names)?
- Show that in general any finite LTS $T$ can be described by using only a sequential fragment of CCS.