Tutorial 4

Exercise 1

Assume an arbitrary CCS defining equation $K \stackrel{\text{def}}{=} P$ where K is a process constant and P is a CCS expression. Prove that $K \sim P$. (Hint: by using SOS rules for CCS, examine the possible transitions from K and P.)

Exercise 2*

Consider the following labelled transition system.

Show that $s \approx t$ by finding a weak bisimulation R containing the pair (s, t).

Exercise 3*

Decide whether the following claims are true or false. Support your claims either by using bisimulation games or directly the definition of strong/weak bisimilarity.

- $a.\tau.Nil \stackrel{?}{\sim} \tau.a.Nil$

- $\tau.a.A + b.B \stackrel{?}{\sim} \tau.(a.A + b.B)$

- $\tau.Nil + (a.Nil \,|\, \bar{a}.Nil) \smallsetminus \{a, b\} \stackrel{?}{\sim} \tau.Nil$

- $a.(\tau.Nil + b.B) \stackrel{?}{\sim} a.Nil + a.b.B$

The same processes but weak bisimilarity instead of the strong one.

- $a.\tau.Nil \stackrel{?}{\approx} \tau.a.Nil$

- $\tau.a.A + b.B \stackrel{?}{\approx} \tau.(a.A + b.B)$

- $\tau.Nil + (a.Nil \,|\, \bar{a}.Nil) \smallsetminus \{a, b\} \stackrel{?}{\approx} \tau.Nil$

- $a.(\tau.Nil + b.B) \stackrel{?}{\approx} a.Nil + a.b.B$

Hint: draw first the LTS generated by the CCS processes.
Home exercise: try to verify your claims by using the tool CWB.

Exercise 4

Prove that for any CCS process P the following law (called idempotency) holds.

- $P + P \sim P$

By using the fact that $\sim \subseteq \approx$ conclude that also $P + P \approx P$.

Exercise 5

In the weak bisimulation game the attacker is allowed to use $\xrightarrow{a}$ moves for the attacks and the defender can use $\xRightarrow{a}$ in response. Argue that if we modify the game rules so that the attacker can also use the long moves $\xRightarrow{a}$ then this does not provide any additional power for the attacker. Conclude that both versions of the game provide the same answer about bisimilarity/nonbisimilarity of two processes.

Exercise 6 (optional)

Define two CCS process constants A and B such that

- A has infinitely many reachable states,

- B has only finitely many reachable states, and

- $A \sim B$.

Challenging continuation of the exercise:
Can you think of a CCS process C with infinitely many reachable states such that there is no CCS process with only finitely many reachable states strongly bisimilar to it? How would you support your claim?

Exercise 7 (optional, easy but recommended)

Consider the tiny communication protocol from Lecture 4.

- Draw the labelled transition system generated by the processes $Spec$ and $Impl$.

- Prove (by hand) that $Spec \approx Impl$. Hint: define a weak bisimulation relation containing $(Spec, Impl)$.

> **If you give me your group solution of this exercise in a written form, I will correct it for you.**