456-358/1: Modelling and Verification (MaV)

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Literature, materials

The course is based on the book

Reactive Systems: Modelling, Specification and Verification

by Luca Aceto, Anna Ingólfsdottir, Kim Guldstrand Larsen, Jiří Srba
Cambridge University Press, August 2007

The authors maintain the web-page

`http://www.cs.aau.dk/rsbook/`

which contains a lot of useful material. (Including the slides kindly provided by Jiří Srba, which serve as a basis of presentations in our course.)

The web-page of our course is

`http://www.cs.vsb.cz/jancar/MOD-VER/mod-ver.htm`

(also reachable from `http://www.cs.vsb.cz/jancar`
and from the KatIS-page of the course).

Focus of the Course

- Study of mathematical models for formal description and analysis of systems (programs).
- Study of formal languages for specification of (properties of) system behaviour.
- Particular focus on parallel and reactive systems.
- Verification tools and implementation techniques underlying them.

Overview of the Course

- Transition systems and CCS.
- Strong and weak bisimilarity, bisimulation games.
- Hennessy-Milner logic and bisimulation.
- Tarski's fixed-point theorem.
- Hennessy-Milner logic with recursively defined formulae.
- Timed CCS.
- Timed automata and their semantics.
- Binary decision diagrams and their use in verification.
- Two mini projects.

Mini Projects

- Verification of a communication protocol in CWB.
- Verification of a real-time algorithm in UPPAAL.

Lectures

- Ask/answer questions. Be active!
- Take your own notes.
- Read the recommended literature as soon as possible after the lecture.

Tutorials/Exercise Sessions

- Supervised peer learning.
- Work in groups of 2 (or 3) people.
- Print out the exercise list, bring literature and your notes.
- Feedback from teaching assistant on your request.
- Star exercises (*) (part of the exam).

Exercise/Project-Credit ("zápočet") and Exam

Exercise/Project-Credit ("zápočet"):

- participating at at least one miniproject (but participating at both is very much desired!) and elaborating a solid respective report,
- for each miniproject you get 10-15 points (or 0).

Exam:

- Individual and oral (the questions will be specified later).
- Preparation time (star exercises).
- Maximum 70 points (necessary minimum 25).

Aims of the Course

Present a general theory of reactive systems and its applications.

- Design.
- Specification.
- Verification (possibly automatic and compositional).

1. Give the students practice in modelling parallel systems in a formal framework.
2. Give the students skills in analyzing behaviours of reactive systems.
3. Introduce algorithms and tools based on the modelling formalisms.

Classical View

Characterization of a Classical Program

Program transforms an input into an output.

- Denotational semantics:
  a meaning of a program is a partial function

$$\mathit{states} \hookrightarrow \mathit{states}$$

- Nontermination is bad!
- In case of termination, the result is unique.

Is this all we need?

Interlude: Verification of a computer program

{ x_1, x_2 are integers satisfying C_1: $x_1 \geq 0, x_2 > 0$ }

Program P

$y_1 := 0; y_2 := x_1;$
  $\{\ x_1 = y_1 x_2 + y_2 \wedge 0 \leq y_2\ \}$ ... INV
while $y_2 \geq x_2$ **do** $(y_1 := y_1 + 1; y_2 := y_2 - x_2);$
$z_1 := y_1; z_2 := y_2$

{ C_2: $x_1 = z_1 x_2 + z_2 \wedge 0 \leq z_2 < x_2$ }

We want to verify: $\{C_1\}P\{C_2\}$ ... (specification of P)

Generated verification conditions:

$\{C_1\}\ y_1 := 0; y_2 := x_1\ \{\mathrm{INV}\}$
$\{\mathrm{INV} \wedge y_2 \geq x_2\}\ y_1 := y_1 + 1; y_2 := y_2 - x_2\ \{\mathrm{INV}\}$
$\{\mathrm{INV} \wedge \neg(y_2 \geq x_2)\}\ z_1 := y_1; z_2 := y_2\ \{C_2\}$

Reactive systems

What about:

- Operating systems?
- Communication protocols?
- Control programs?
- Mobile phones?
- Vending machines?

Reactive systems

Characterization of Reactive Systems

Reactive System is a system that computes by reacting to stimuli from its environment.

Key Issues:

- communication and interaction
- parallelism

Nontermination is good!

The result (if any) does not have to be unique.

Analysis of Reactive Systems

Questions

- How can we develop (design) a system that "works"?
- How do we analyze (verify) such a system?

Fact of Life

Even short parallel programs may be hard to analyze.

Example: Peterson's protocol

Concurrent, parallel, interactive, 'nondeterministic' systems, with ongoing behaviour ...
No input-output characterization (specification) ...
Verification of 'simple' properties ...

Peterson's protocol (to avoid critical section clash)

Process A:

** noncritical region **
$flag_A := true$
$turn := B$
waitfor
$(flag_B = false \vee turn = A)$
** critical region **
$flag_A := false$
** noncritical region **

Process B:

** noncritical region **
$flag_B := true$
$turn := A$
waitfor
$(flag_A = false \vee turn = B)$
** critical region **
$flag_B := false$
** noncritical region **

The Need for a Theory

Conclusion

We need formal/systematic methods (tools), otherwise ...

- Intel's Pentium-II bug in floating-point division unit
- Ariane-5 crash due to a conversion of 64-bit real to 16-bit integer
- Mars Pathfinder
- ...

Classical vs. Reactive Computing

	Classical	Reactive/Parallel
interaction	no	yes
nontermination	undesirable	often desirable
unique result	yes	no
semantics	$states \hookrightarrow states$	**?**

How to Model Reactive Systems

Question

What is the most abstract view of a reactive system (process)?

Answer

A process performs an action and becomes another process.

Labelled Transition System

Definition

A labelled transition system (LTS) is a triple $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$ where

- $Proc$ is a set of states (or processes),
- Act is a set of labels (or actions), and
- for every $a \in Act$, $\xrightarrow{a} \subseteq Proc \times Proc$ is a binary relation on states called the transition relation.

We will use the infix notation $s \xrightarrow{a} s'$ meaning that $(s, s') \in \xrightarrow{a}$.

Sometimes we distinguish the initial (or start) state.

Interlude: Binary Relations

Definition

A binary relation R on a set A is a subset of $A \times A$.

$$R \subseteq A \times A$$

Sometimes we write $x \; R \; y$ instead of $(x, y) \in R$.

Some properties of relations

- R is reflexive if $(x, x) \in R$ for all $x \in A$
- R is symmetric if $(x, y) \in R$ implies that $(y, x) \in R$ for all $x, y \in A$
- R is transitive if $(x, y) \in R$ and $(y, z) \in R$ implies that $(x, z) \in R$ for all $x, y, z \in A$

Closures

Let R, R' and R'' be binary relations on a set A.

Reflexive Closure

R' is the reflexive closure of R if and only if

1. $R \subseteq R'$,
2. R' is reflexive, and
3. R' is the *smallest* relation that satisfies the two conditions above, which means the following:
   for any relation R'', if $R \subseteq R''$ and R'' is reflexive then $R' \subseteq R''$.

Let R, R' and R'' be binary relations on a set A.

Symmetric Closure

R' is the symmetric closure of R if and only if

1. $R \subseteq R'$,
2. R' is symmetric, and
3. R' is the *smallest* relation that satisfies the two conditions above.

Closures

Let R, R' and R'' be binary relations on a set A.

Transitive Closure

R' is the transitive closure of R if and only if

1. $R \subseteq R'$,
2. R' is transitive, and
3. R' is the *smallest* relation that satisfies the two conditions above.

Labelled Transition Systems – Notation

Let $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$ be an LTS.

- we extend $\xrightarrow{a}$ to the elements of Act^*
- $\longrightarrow = \bigcup_{a \in Act} \xrightarrow{a}$
- $\longrightarrow^*$ is the reflexive and transitive closure of $\longrightarrow$
- $s \xrightarrow{a}$ and $s \stackrel{a}{\not\longrightarrow}$
- reachable states

How to Describe LTS?

Syntax		Semantics
unknown entity	$\longrightarrow$	known entity
programming language	$\longrightarrow$	what (denotational) or how (operational) it computes
???	$\longrightarrow$	Labelled Transition Systems

CCS

Calculus of Communicating Systems

CCS

Process algebra called "Calculus of Communicating Systems".

Insight of Robin Milner (1989)

Concurrent (parallel) processes have an algebraic structure.

$$\boxed{P_1}\ op\ \boxed{P_2} \Rightarrow \boxed{P_1\ op\ P_2}$$

Process Algebra

Basic Principle

1. Define a few atomic processes (modelling the simplest process behaviour).
2. Define compositionally new operations (building more complex process behaviour from simple ones).

Example

1. atomic instruction: assignment (e.g. x:=2 and x:=x+2)
2. new operators:
   - sequential composition ($P_1; P_2$)
   - parallel composition ($P_1 \parallel P_2$)

   Now e.g. (x:=1 $\parallel$ x:=2); x:=x+2; (x:=x-1 $\parallel$ x:=x+5) is a process.

A CCS Process: Black-Box View

What is a CCS Process to its Environment?

A CCS process is a computing agent that may communicate with its environment via its interface.

Interface = Collection of communication ports/channels, together with an indication of whether used for input or output.

Example: A Computer Scientist

Process interface:

- coffee (input port)
- $\overline{\text{coin}}$ (output port)
- $\overline{\text{pub}}$ (output port)

Question: How do we describe the behaviour of the "black-box"?

CCS Basics (Sequential Fragment)

- *Nil* (or 0) process (the only atomic process)
- action prefixing ($a.P$)
- names and recursive definitions ($\stackrel{\mathrm{def}}{=}$)
- nondeterministic choice ($+$)

This is Enough to Describe Sequential Processes

Any finite LTS can be (up to isomorphism) described by using the operations above.