## **Tutorial 6**

## Exercise 1\*

Consider the set  $\{a, b, c\}$  (with three elements). Define some nontrivial function  $f : 2^{\{a, b, c\}} \to 2^{\{a, b, c\}}$  which is monotonic.

- Compute the greatest fixed point of f by using directly the Tarski's fixed point theorem.
- Compute the least fixed point of f by starting from  $\emptyset$  and applying repeatedly the function f until the fixed point is reached.

## **Exercise 2**

Consider the following labelled transition system.

$$s \xleftarrow{b}{s_1 \xrightarrow{b}{b}} s_2^a$$

Compute for which sets of states  $\llbracket X \rrbracket \subseteq \{s, s_1, s_2\}$  the following formulae are true.

- $X = \langle a \rangle t t \lor [b] X$
- $X = \langle a \rangle t t \lor ([b] X \land \langle b \rangle t)$

## Exercise 3\*

Consider the following labelled transition system.



Using the game characterization for recursive Hennessy-Milner formulae decide whether the following claims are true or false and discuss what properties the formulae describe:

- $s \stackrel{?}{\models} X$  where  $X \stackrel{\min}{=} \langle c \rangle t \lor \langle Act \rangle X$
- $s \models^? X$  where  $X \stackrel{\min}{=} \langle c \rangle t t \lor [Act] X$
- $s \models X$  where  $X \stackrel{\text{max}}{=} \langle b \rangle X$
- $s \stackrel{?}{\models} X$  where  $X \stackrel{\max}{=} \langle b \rangle t t \wedge [a] X \wedge [b] X$