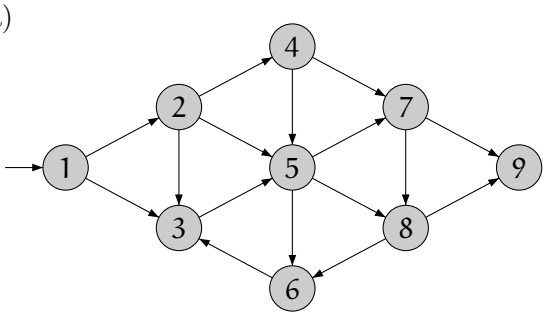
Tutorial 6

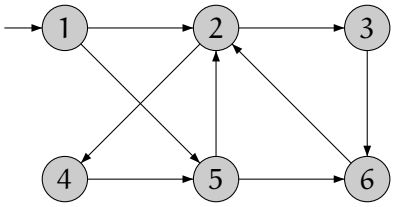
Exercise 1: Recall the formulation Generalized Geography problem.

Determine for the following two instances of Generalized Geography, which of the players has a winning strategy in the given game:

a)



b)



Exercise 2: Explain why each PSPACE-hard problem is also NP-hard.

Exercise 3: Consider those sequences of word $w_1, w_2, \ldots, w_k$ where all words are of the same length and where every two consecutive words differ in exactly one symbol.

Example:

head, hear, near, fear, bear, beer, deer, deed, feed, feet, fret, free

Show that the following problem is in PSPACE:

INPUT: A deterministic finite automaton $\mathcal{A}$ and a pair of words u and v of the same length.

QUESTION: Is there a sequences of the type described above starting with word u and ending with word v such that every word in this sequence is accepted by automaton $\mathcal{A}$?

Exercise 4: Boolean formulas φ and ψ are ***equivalent*** if they have the same truth value for each truth valuation.

The notation $|\varphi|$ represents the size of formula φ, i.e., the number of symbols of this formula.

A formula φ is called ***minimal*** of for each formula ψ that is equivalent to formula φ, we have $|\psi| \geq |\varphi|$.

Show that the following problem is in PSPACE:

INPUT: Boolean formula φ.

QUESTION: Is formula φ minimal?

Does this problem belong to NP or co-NP class?

Exercise 5: Consider the following game played by two players on an undirected graph G. One player plays with a figure representing a cat and the other with a figurer representing a mouse. These figures are put on nodes of the graph. Players alternate in moves. Player who is on turn, takes her/his figure and moves it to a neighbouring node. One of nodes of the graph is denoted as a "mouse hole". A player with the cat can not go to this node. A player with the cat wins if the figures of the cat and the mouse occur on the same node of the graph. A player with the mouse wins if she/he succeeds in reaching the node representing the "mouse hole" with her/his figure.

Show that the following problem is in P:

INPUT: An undirected graph G with denoted positions of the cat and the mouse, and with one node representing the mouse hole, and information, whose turn it is.

QUESTION: Does the player with the cat a winning strategy in the given position?