Tutorial 1

Exercise 1: For every of the following formal descriptions of sets describe (in your own words) what elements the given set contains:

a) $\{1, 3, 5, 7, \dots\}$
b) $\{\dots, -4, -2, 0, 2, 4, \dots\}$
c) $\{n \mid n = 2m \text{ for some } m \in \mathbb{N}\}$
d) $\{n \mid n = 2m \text{ for some } m \in \mathbb{N} \text{ and } n = 3k \text{ for some } k \in \mathbb{N}\}$
e) $\{n \in \mathbb{Z} \mid n = n + 1\}$

Exercise 2: Write formal descriptions of the followning sets:

a) The set containing the numbers 1, 10, and 100.
b) The set containing all integers that are greater than 5.
c) The set containing all natural numbers that are less than 5.
d) The set containing no elements.
e) The set containing all subsets of a given set X.

Exercise 3: Consider sets $A = \{x, y, z\}$ and $B = \{x, y\}$.

a) Is $A \subseteq B$?
b) Is $A \supseteq B$?
c) What is $A \cup B$?
d) What is $A \cap B$?
e) What is $A \times B$?
f) What is $\mathcal{P}(B)$?

Exercise 4: Decide whether the following holds:

a) $a \in \{\{a\}, \{a, \{a\}\}\}$
b) $\{a, \{a\}\} \cap \mathcal{P}(\{a, \{a\}\}) = \emptyset$
c) $\{\emptyset\} \in \{\{\emptyset\}\}$

Exercise 5: List all elements of these sets:

a) $\{a, \{a\}\} \cup \{a, \{b\}, c\}$
b) $\{a, \{a\}\} \cap \{a, \{b\}, c\}$
c) $\{a, \{a\}\} - \{a, \{b\}, c\}$

Exercise 6: If a set A has a elements and a set B has b elements, how many elements are in $A \times B$? Explain your answer.

Exercise 7: If a set C has c elements, how many elements are in the set $\mathcal{P}(C)$? Explain your answer.

Exercise 8: Recall the notion of a relation, and what types of relations do you know (homogenous vs. non-homogenous, unary, binary, etc.).

a) Define precisely, what it means that a relation is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, transitive, functional.

b) Consider the connection between binary relations and directed graphs (which can be infinite) and describe what the individual properties from the previous point mean from the point of view of the graph representing a given relation.

c) Recall what it means that a relation is an equivalence. How is the notion of an equivalence related to the notion of a partition?

Exercise 9: For each part, give an example of a binary relation that satisfies the condition:

a) Reflexive and symmetric but not transitive.
b) Reflexive and transitive but not symmetric.
c) Symmetric and transitive but not reflexive.

Exercise 10: Recall what it means that a relation is an order. What kinds of orders do you know? Give some example of an order, defined on the set of natural numbers, that is not a total order.

Exercise 11: Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{6, 7, 8, 9, 10\}$. The unary function $f : X \to Y$ and the binary function $g : X \times Y \to Y$ are described in the following tables:

n	$f(n)$
1	6
2	7
3	6
4	7
5	6

g	6	7	8	9	10
1	10	10	10	10	10
2	7	8	9	10	6
3	7	7	8	8	9
4	9	8	7	6	10
5	6	6	6	6	6

a) What is the value of $f(2)$?
b) What are the domain and range of function f?
c) What is the value of $g(2, 10)$?
d) What are the domain and range of function g?

e) What is the value of $g(4, f(4))$?

Exercise 12: Recall what it means that a function is injective, surjective, and bijective. Is the function $f(x) = x + 1$ injective, surjective, and/or bijective on the set of natural numbers $\mathbb{N}$? And on the set of integers $\mathbb{Z}$?

Exercise 13: Recall the notion of a binary operation on a set and what it means that a given operation is associative, and what it means that it is commutative. Give an example of an operation that:

a) is associative but is not commutative,
b) is commutative but is not associative.
c) is not associative nor commutative.