

## Tutorial 4

**Exercise 1:** Consider the following context-free grammar:

$$\begin{aligned} S &\longrightarrow aBb \mid AB \\ A &\longrightarrow bAb \mid a \\ B &\longrightarrow \varepsilon \mid aABb \end{aligned}$$

- Give (some) derivation of word **babaab** in this grammar.
- Draw the corresponding derivation tree.
- Write the left and right derivations corresponding to the derivation tree drawn in the previous point.

**Exercise 2:** Construct context-free grammars for all following languages:

- $L_1 = \{w \in \{a, b, c\}^* \mid w \text{ contains subword } babb\}$
- $L_2 = \{0^n 1^m \mid 1 \leq n < m\}$
- $L_3 = \{a^n b^m a^{n+2} \mid m, n \in \mathbb{N}\}$
- $L_4 = \{w \in \{0, 1\}^* \mid w = w^R\}$
- $L_5 = \{w \in \{0, 1\}^* \mid |w|_0 > 1, |w|_1 \leq 2\}$
- $L_6 = \{0^n w w^R 1^n \mid w \in \{0, 1\}^*, n \in \mathbb{N}\}$
- $L_7 = \{w \in \{a, b\}^* \mid \text{in } w, \text{ every } a \text{ is directly followed by } b, \text{ or } w = b^n a^m, \text{ where } 0 \leq m \leq n\}$
- $L_8 = \{uv^Rv \mid u, v \in \{0, 1\}^*, |u|_0 \bmod 4 = 2, u \text{ ends with suffix } 101 \text{ and } v \text{ contains subword } 10\}$
- $L_9 = \{w \in \{a, b\}^* \mid w = w^R, |w| \bmod 4 = 0\}$
- $L_{10} = \{w \in \{a, b\}^* \mid w = w^R, |w| \bmod 3 = 0\}$
- $L_{11} = \{w \in \{a, b, c\}^* \mid \text{every sequence of } a\text{'s is directly followed by a sequence of } b\text{'s, which is twice as long}\}$
- $L_{12} = \{w \in \{0, 1\}^* \mid |w|_0 = |w|_1\}$

**Exercise 3:** Decide for the following pairs of grammars if both grammars generate the same language. Justify your answers.

- $S \longrightarrow aaSbb \mid ab \mid aabb$ 
 $S \longrightarrow aSb \mid ab$
- $S \longrightarrow aaSbb \mid ab \mid \varepsilon$ 
 $S \longrightarrow aSb \mid ab$
- $S \longrightarrow aaSb \mid ab \mid \varepsilon$ 
 $S \longrightarrow aSb \mid aab \mid \varepsilon$

**Exercise 4:** Construct a context-free grammar for the language  $L$  over the alphabet  $\Sigma = \{(, ), [, ]\}$  consisting of all “correctly parenthesized” expressions. As correctly parenthesized

expressions we consider those sequences of symbols where each left parenthesis has a corresponding right parenthesis of the same type, and where parenthesis do not “cross” (i.e., corresponding pairs of parenthesis are composed correctly).

**Exercise 5:** Is the following grammar unambiguous?

$$\begin{aligned} E &\longrightarrow E + E \mid F \\ F &\longrightarrow (E) \mid F \times F \mid \mathbf{a} \end{aligned}$$

**Exercise 6:** Propose a syntax for writing simple arithmetic expressions as words over the alphabet

$$\Sigma = \{A, B, \dots, Z, a, b, \dots, z, 0, 1, \dots, 9, ., +, -, *, /, (, )\}.$$

and describe the proposed syntax by a context-free grammar.

**Exercise 7:** Construct a context-free grammar generating the set of all well-formed formulas of the propositional logic. Consider the set  $At = \{x_0, x_1, x_2, \dots\}$  as the set of atomic propositions, where individual variables can be written as  $x_0, x_1, x_2, \dots$

- a) Find out if the grammar you have constructed is unambiguous.
- b) If the grammar is ambiguous then modify it to be unambiguous.
- c) Modify your grammar in such a way, which ensures that a structure of a derivation tree for an arbitrary derivation in the grammar reflects the “real” priority of logical connectives, i.e.  $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$  (from the highest to the lowest).