Tutorial 6

Exercise 1: Construct a Turing machine that erases from a given word over alphabet $\{a, b\}$ the longest possible sequences of a's of the same length from its start and its end. (From word '$aaababaa$' it makes '$abab$', and from '$aaabab$' it does not erase anything. From 'aaa' it makes ε.)

Exercise 2: Construct a Turing machine that from a given word over alphabet $\{a, b, c\}$ removes all occurrences of symbol a.

Exercise 3: Construct Turing machines recognizing the following languages:

a) $\{ w \in \{a, b\}^* \mid w = w^R \}$

b) $\{ wcw \mid w \in \{a, b\}^* \}$

Remark: The alphabet is $\{a, b, c\}$.

c) $\{ ww \mid w \in \{a, b\}^* \}$

Exercise 4: Construct a Turing machine that divides a given number written in binary by three.

For example, for input 100101 (which, written in decimal, is 37), the output will be the string 1100 (which, written in decimal, is 12), since $\lfloor 37/3 \rfloor = 12$.

Hint: Recall the standard elementary school algorithm for division of numbers and proceed directly according to this algorithm.

Exercise 5: Describe how a Turing machine with one-side infinite tape can simulate a computation of a Turing machine with a tape infinite on both sides.