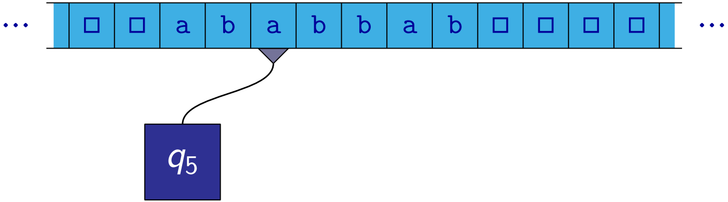
Turing Machines

Turing Machine

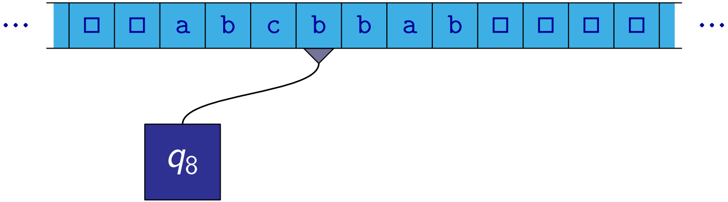
Turing machine — a device similar to a finite automaton with the following differences:

- the head can move in both directions
- it is possible to write on a current position of the head
- the tape is infinite



Turing machine — a device similar to a finite automaton with the following differences:

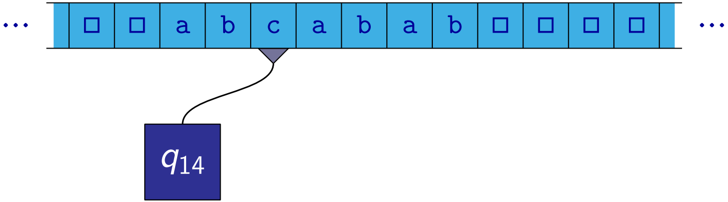
- the head can move in both directions
- it is possible to write on a current position of the head
- the tape is infinite



Turing Machine

Turing machine — a device similar to a finite automaton with the following differences:

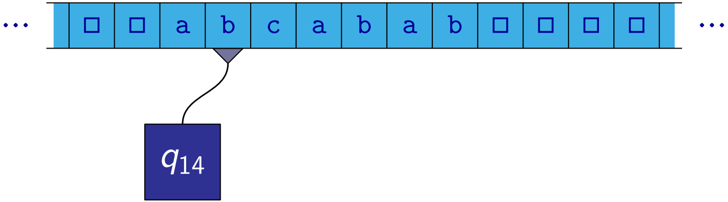
- the head can move in both directions
- it is possible to write on a current position of the head
- the tape is infinite



Turing Machine

Turing machine — a device similar to a finite automaton with the following differences:

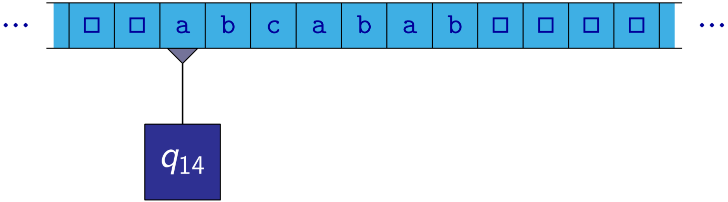
- the head can move in both directions
- it is possible to write on a current position of the head
- the tape is infinite



Turing Machine

Turing machine — a device similar to a finite automaton with the following differences:

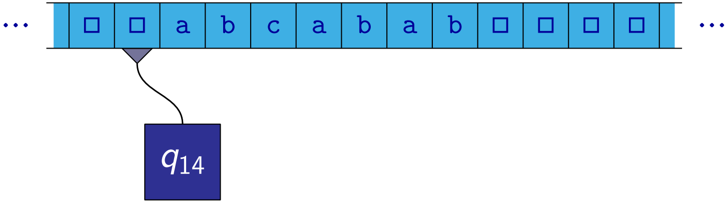
- the head can move in both directions
- it is possible to write on a current position of the head
- the tape is infinite



Turing Machine

Turing machine — a device similar to a finite automaton with the following differences:

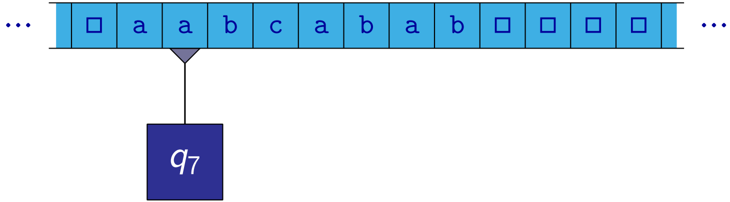
- the head can move in both directions
- it is possible to write on a current position of the head
- the tape is infinite



Turing Machine

Turing machine — a device similar to a finite automaton with the following differences:

- the head can move in both directions
- it is possible to write on a current position of the head
- the tape is infinite



Turing Machine

Alan M. Turing, "On Computable Numbers, with an application to the Entscheidungsproblem", *Proceedings of the London Mathematical Society*, 42 (1936), pp. 230–265, Erratum: Ibid., 43 (1937), pp. 544–546.

Turing Machine

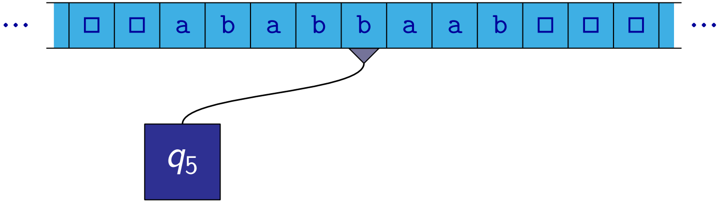
Definition

Formally, **Turing machine** is defined as a tuple $\mathcal{M} = (Q, \Sigma, \Gamma, \delta, q_0, F)$ where:

- Q is a finite non-empty set of **states**
- Γ is a finite (non-empty) set of **tape symbols** (**tape alphabet**)
- $\Sigma \subseteq \Gamma$ is a finite non-empty set of **input symbols** (**input alphabet**)
- $\delta : (Q - F) \times \Gamma \to Q \times \Gamma \times \{-1, 0, +1\}$ is a **transition function**
- $q_0 \in Q$ is an **initial state**
- $F \subseteq Q$ is a set of **final states**

We assume that $\Gamma - \Sigma$ always contains a special element $\square$ denoting a **blank** symbol.

Configurations of a Turing Machine

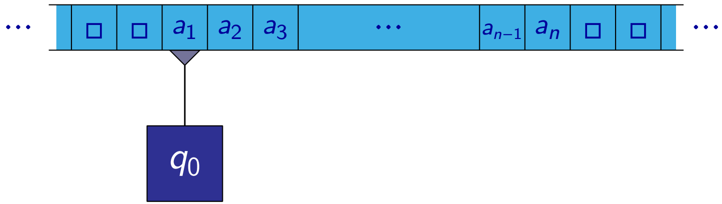


A configuration of a Turing machine is given by:

- a state of its control unit
- a content of the tape
- a position of the head

Configurations of a Turing Machine

A computation of a Turing machine $\mathcal{M} = (Q, \Sigma, \Gamma, \delta, q_0, F)$ over a word $w \in \Sigma^*$,
where $w = a_1 a_2 \cdots a_n$, starts in an **initial configuration**:



- the state of the control unit is q_0
- word w is written on the tape, remaining cells of the tape are filled with the blank symbols ($\square$)
- the head is on the first symbol of the word w (or on symbol $\square$ when $w = \varepsilon$)

Turing Machine

One step of a Turing machine:

Let us assume that:

- the state of the control unit is q
- the cell of the tape on the position of the head contains symbol b

Let us say that $\delta(q, b) = (q', b', d)$ where $d \in \{-1, 0, +1\}$.

One step of the Turing machine is performed as follows:

- the state of the control unit is changed to q'
- symbol b' is written on the tape cell on the position of the head instead of b
- The head is moved depending on d:
  - for $d = -1$ the head is moved one cell left
  - for $d = +1$ the head is moved one cell right
  - for $d = 0$ the position of the head is not changed

Turing Machine

- A Turing machine performs these steps until a state of its control unit is a state from the set F.
- Those configurations where a state of the control unit belongs to set F are **final configurations**.
- A computation ends in a final configuration.
- A computation of a machine $\mathcal{M}$ over a word w can be infinite.

Turing Machine

We often choose the set of final states $F = \{q_{acc}, q_{rej}\}$.

Then we can define for a word $w \in \Sigma^*$ if a given Turing machine accepts it:

- If the state of the control unit after the computation over the word w is q_{acc}, the machine accepts the word w.
- If the state of the control unit after the computation over the word w is q_{rej}, the machine does not accept the word w.
- The computation of the machine over the word w can be infinite. In this case the machine does not accept the word w.

The language $\mathcal{L}(\mathcal{M})$ of a Turing machine $\mathcal{M}$ is the set of all words accepted by $\mathcal{M}$.

Turing Machine

A language $L \subseteq \Sigma^*$ is **accepted** by a Turing machine $\mathcal{M}$ if:

- for each word $w \in \Sigma^*$ it holds that $w \in L$ iff the computation of $\mathcal{M}$ over w ends in final state q_{acc}.

(So computations over words that do not belong to L can end in state q_{rej} or be infinite.)

Language $L \subseteq \Sigma^*$ is **recognized** by a Turing machine $\mathcal{M}$ if:

- for each word $w \in L$ the computation of machine $\mathcal{M}$ over w ends in final state q_{acc}.
- for every word $w \in (\Sigma^* - L)$ the computation of machine $\mathcal{M}$ over w ends in final state q_{rej}.

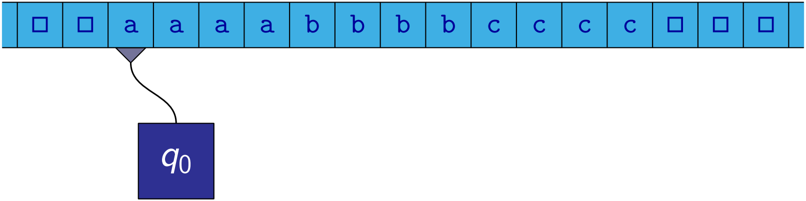
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



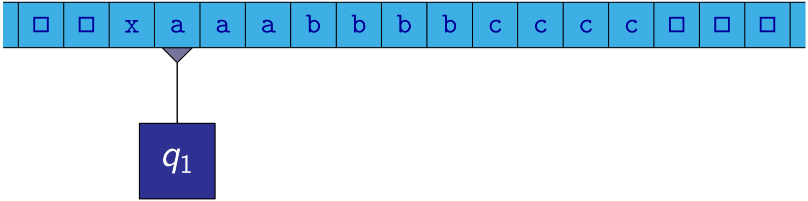
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



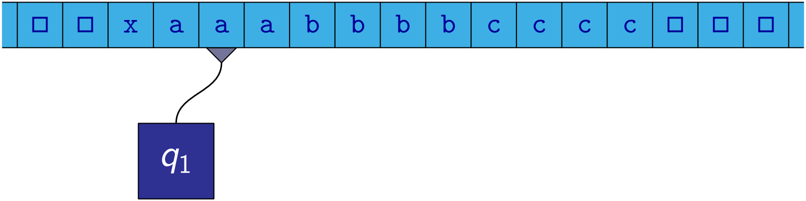
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



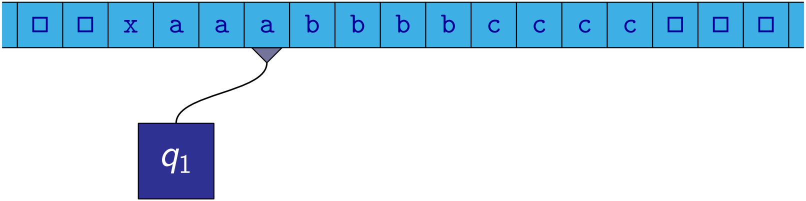
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



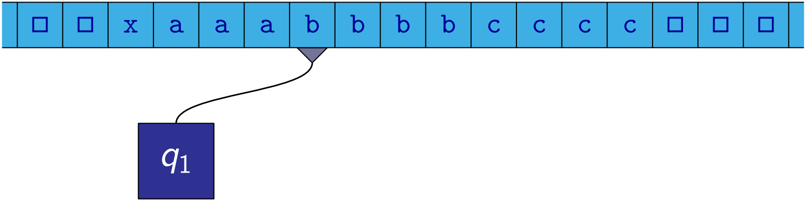
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



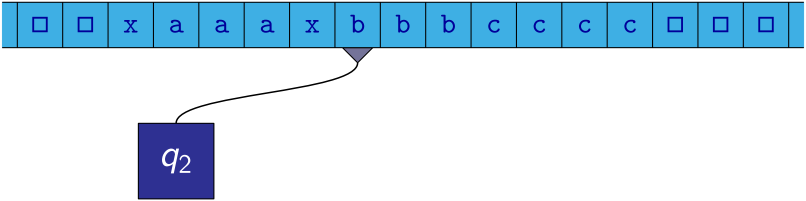
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



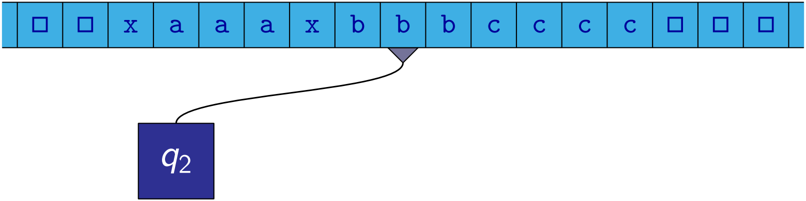
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



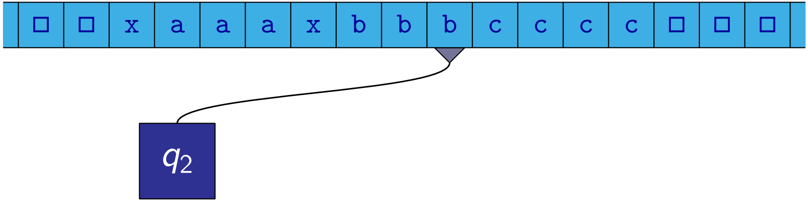
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



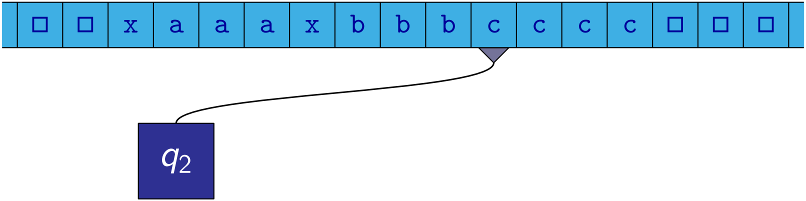
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



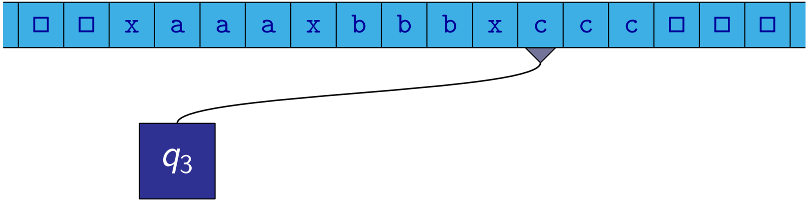
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



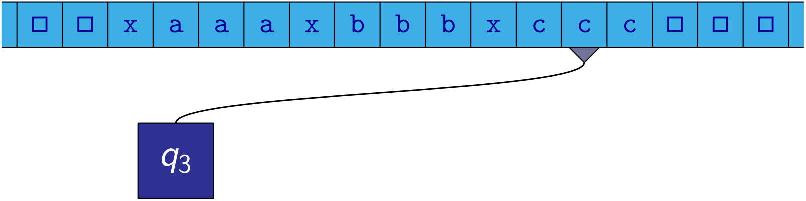
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



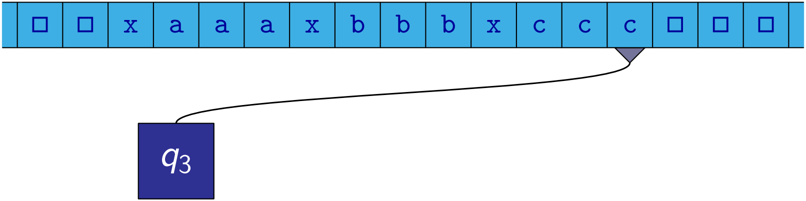
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



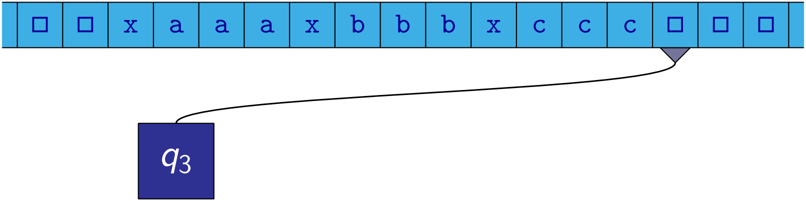
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



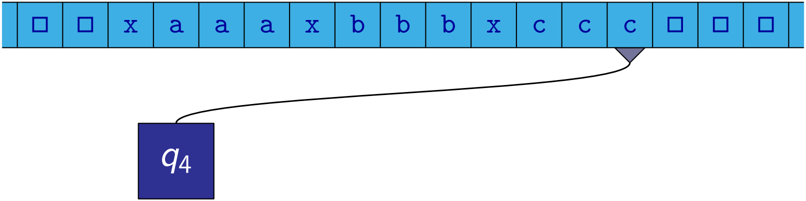
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\Box, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\Box$	a	b	c	x
q_0	$(q_{acc}, \Box, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \Box, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \Box, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \Box, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \Box, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



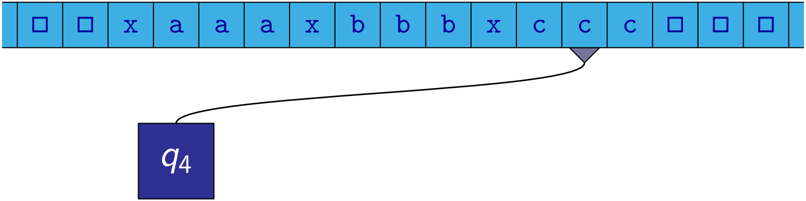
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



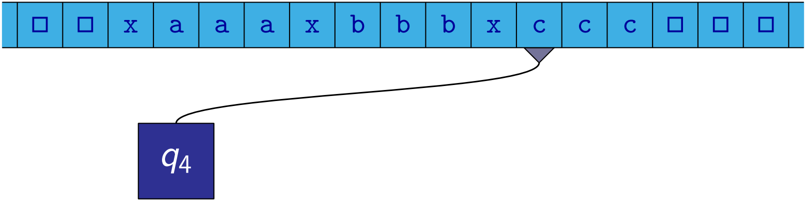
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



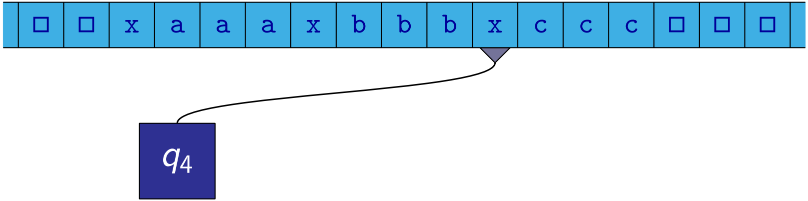
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



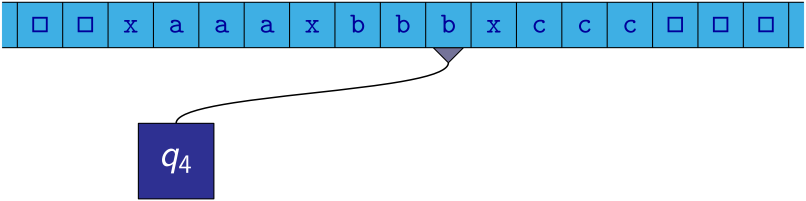
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$

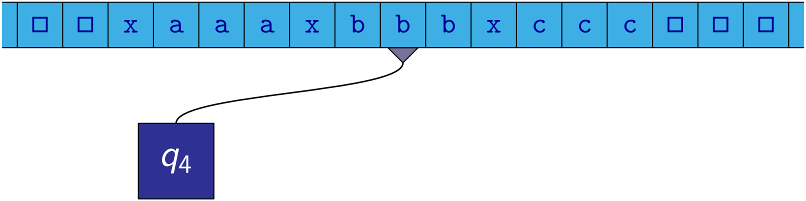


Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



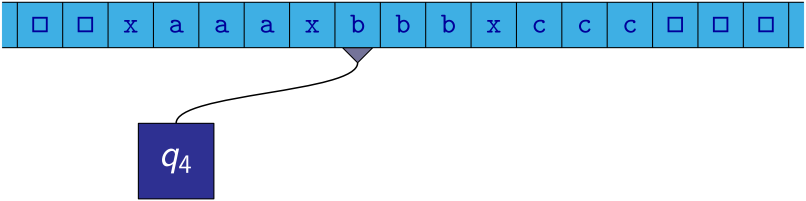
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



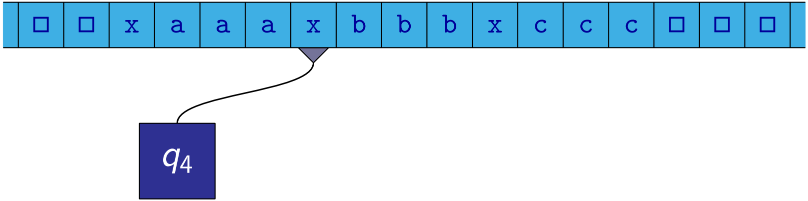
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



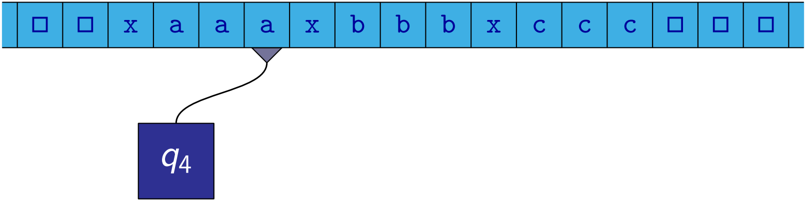
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



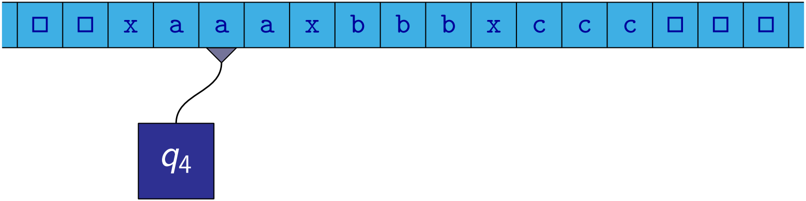
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



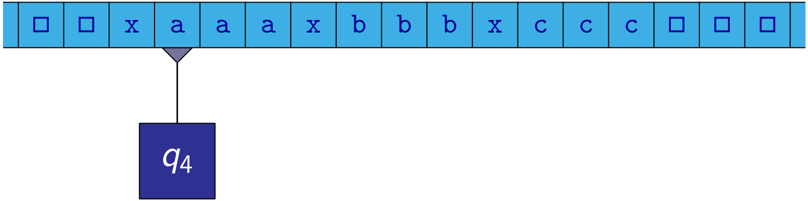
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$

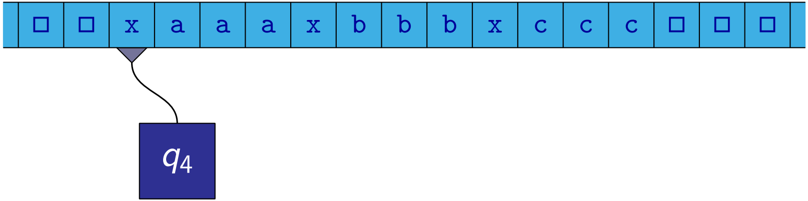


Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



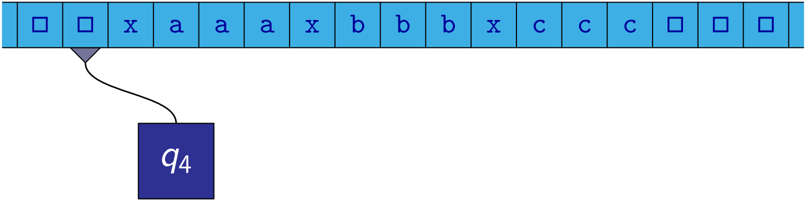
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



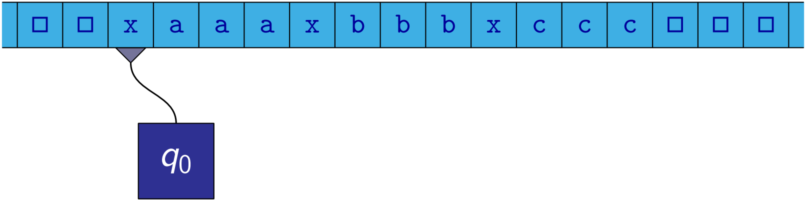
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$

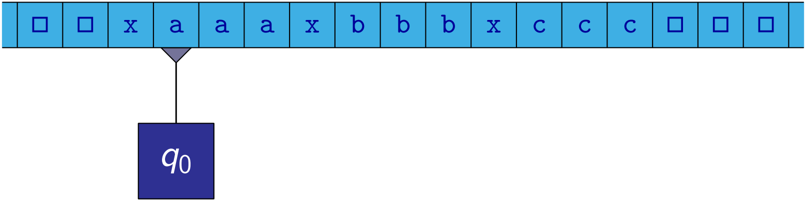


Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



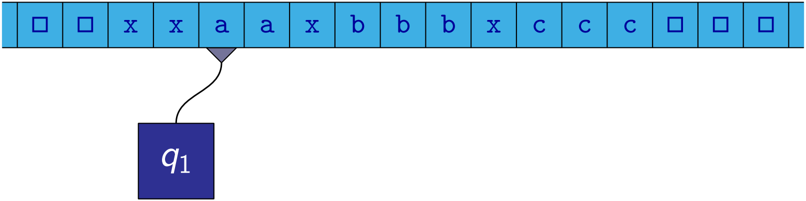
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



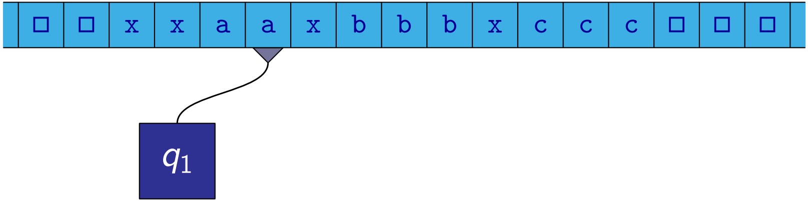
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad\quad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad\quad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



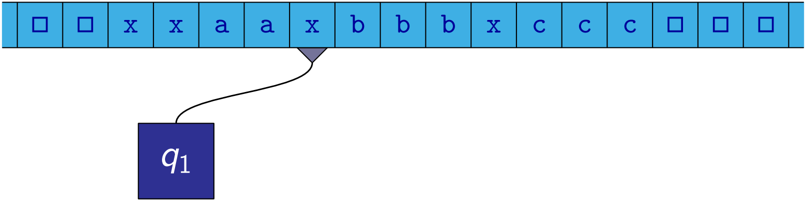
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\texttt{a}, \texttt{b}, \texttt{c}\}$ $\qquad \Gamma = \{\square, \texttt{a}, \texttt{b}, \texttt{c}, \texttt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \texttt{x}, +1)$	$(q_{rej}, \texttt{b}, 0)$	$(q_{rej}, \texttt{c}, 0)$	$(q_0, \texttt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \texttt{a}, +1)$	$(q_2, \texttt{x}, +1)$	$(q_{rej}, \texttt{c}, 0)$	$(q_1, \texttt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \texttt{a}, 0)$	$(q_2, \texttt{b}, +1)$	$(q_3, \texttt{x}, +1)$	$(q_2, \texttt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \texttt{a}, 0)$	$(q_{rej}, \texttt{b}, 0)$	$(q_3, \texttt{c}, +1)$	$(q_3, \texttt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \texttt{a}, -1)$	$(q_4, \texttt{b}, -1)$	$(q_4, \texttt{c}, -1)$	$(q_4, \texttt{x}, -1)$



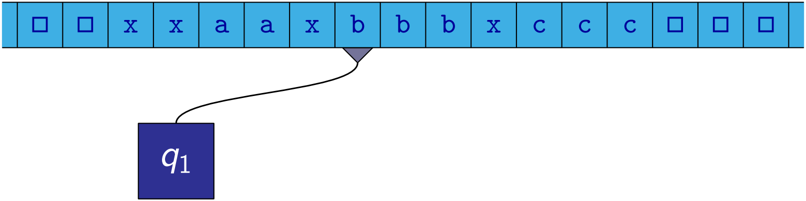
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



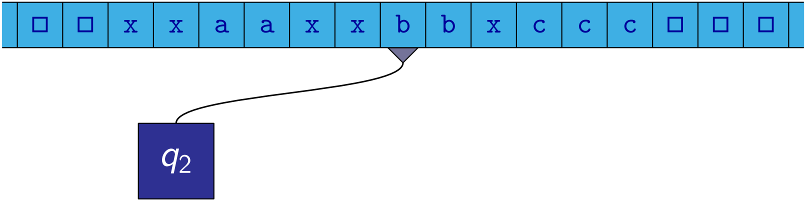
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\Box, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\Box$	a	b	c	x
q_0	$(q_{acc}, \Box, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \Box, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \Box, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \Box, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \Box, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$

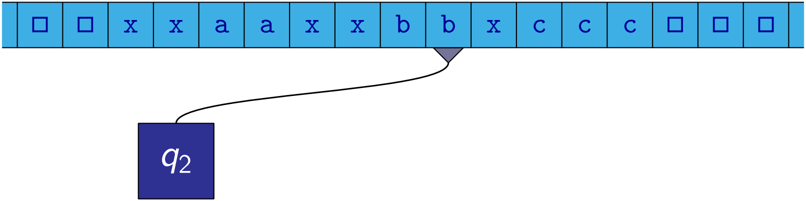


Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\Gamma = \{\Box, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\Box$	a	b	c	x
q_0	$(q_{acc}, \Box, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \Box, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \Box, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \Box, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \Box, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



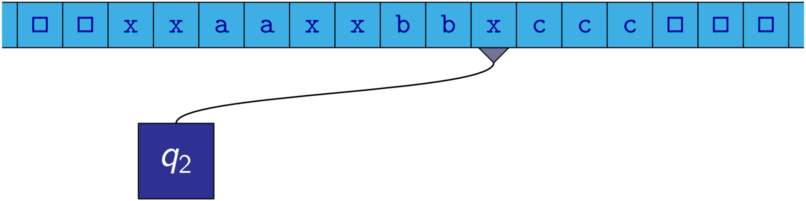
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



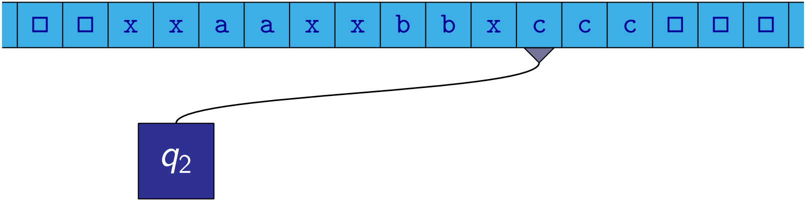
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



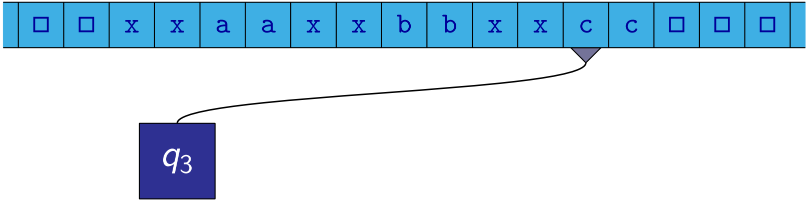
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



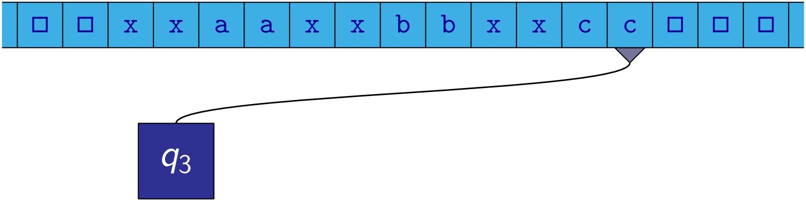
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



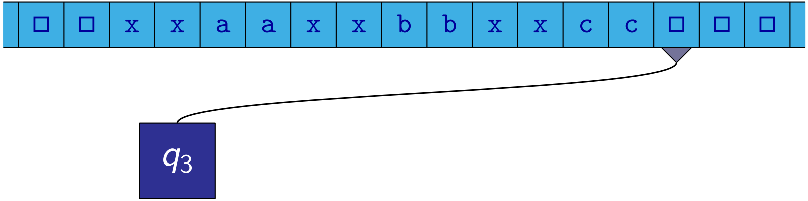
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



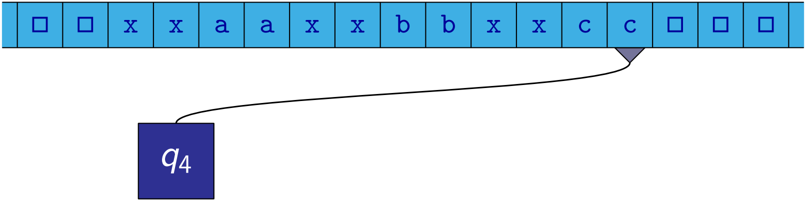
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



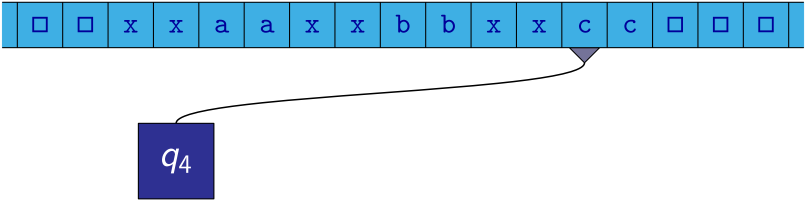
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\Box, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\Box$	a	b	c	x
q_0	$(q_{acc}, \Box, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \Box, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \Box, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \Box, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \Box, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



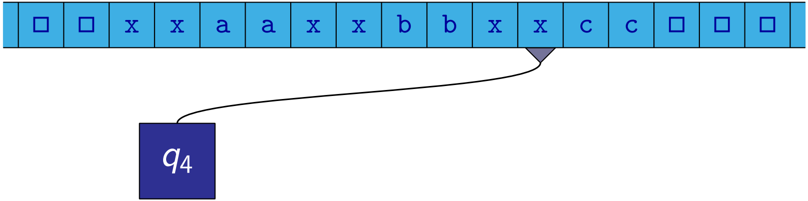
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



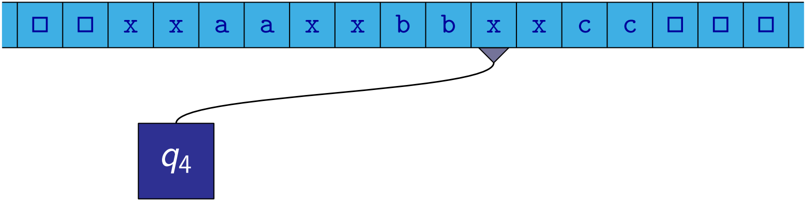
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



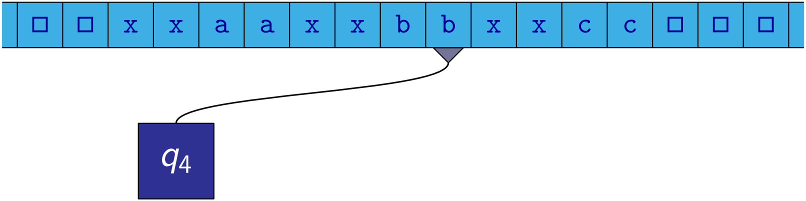
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



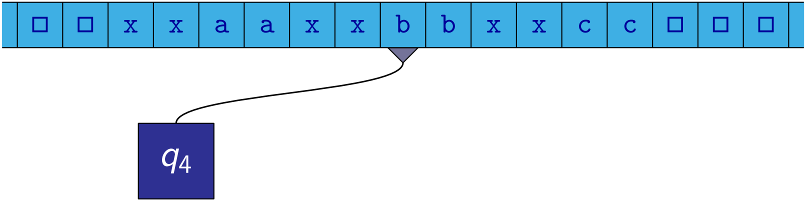
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



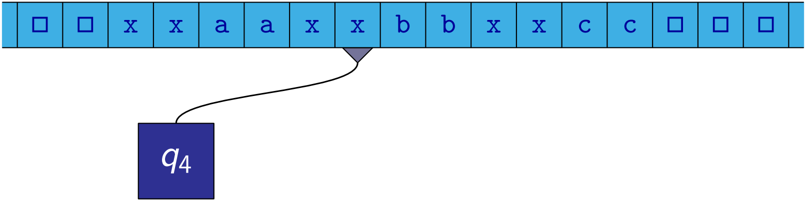
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



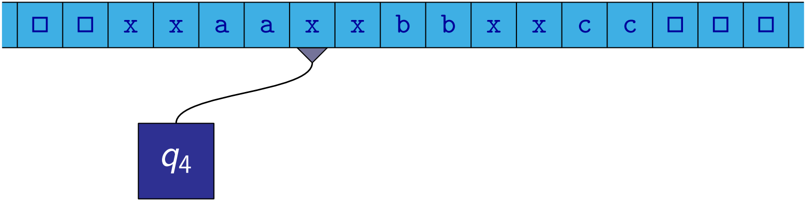
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



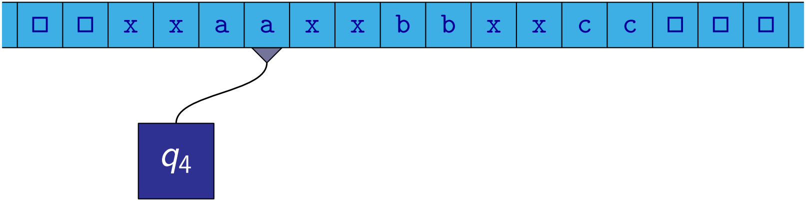
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



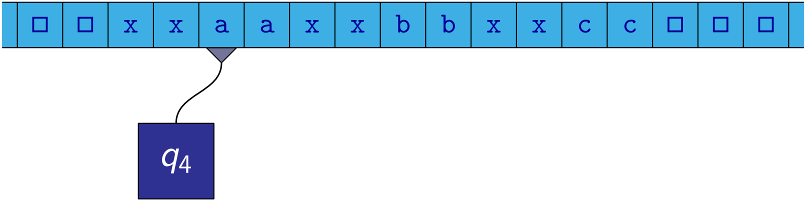
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



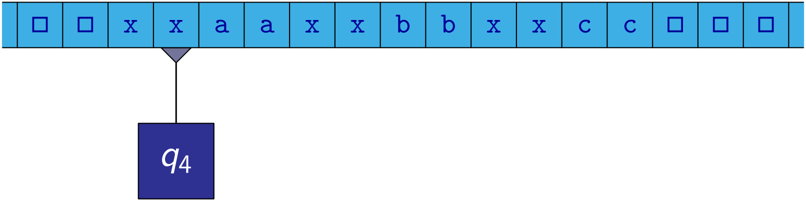
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



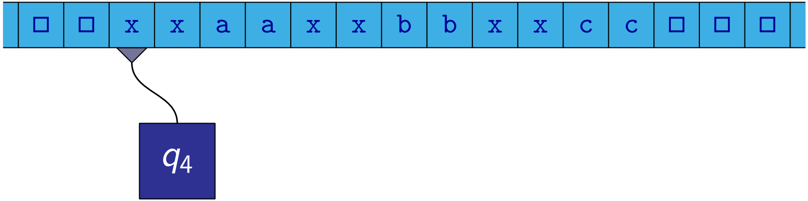
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



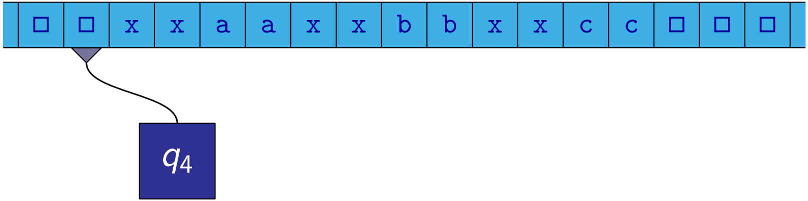
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



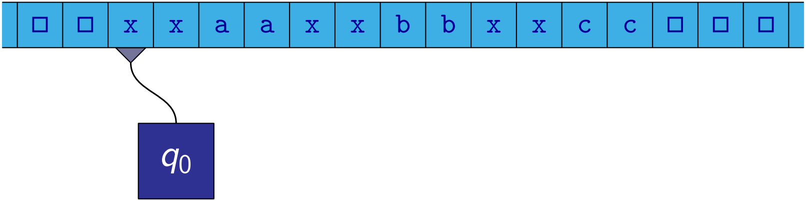
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



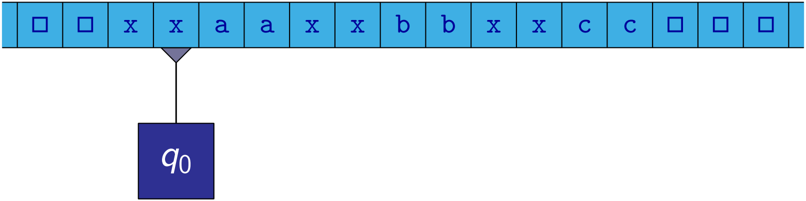
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



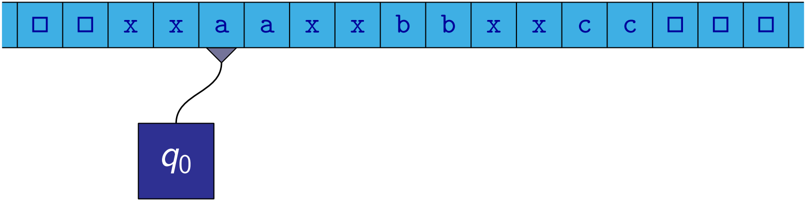
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



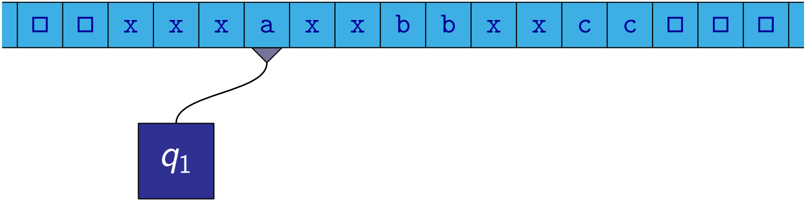
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



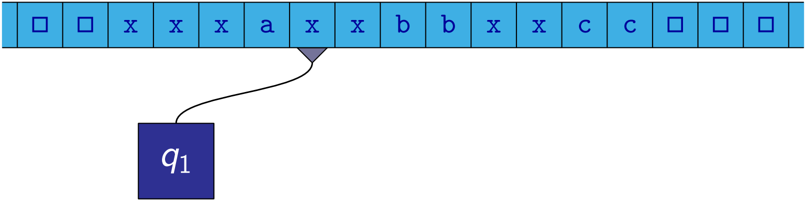
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



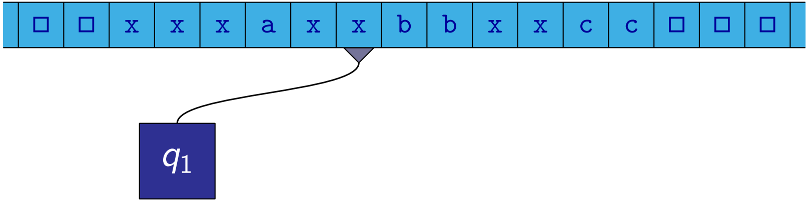
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	$\mathtt{a}$	$\mathtt{b}$	$\mathtt{c}$	$\mathtt{x}$
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



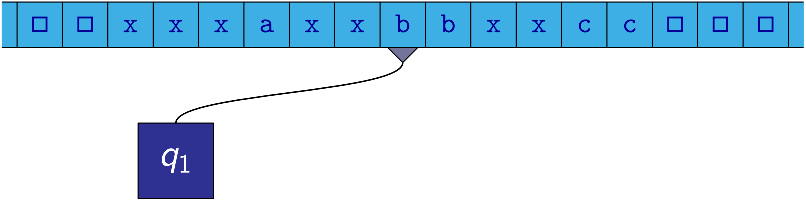
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



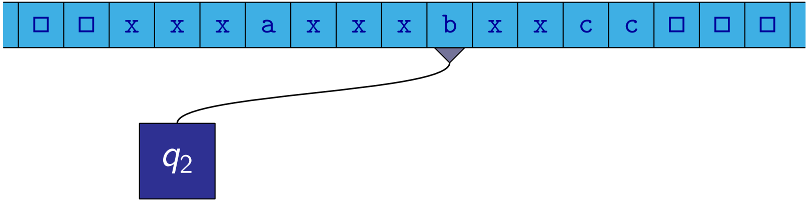
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



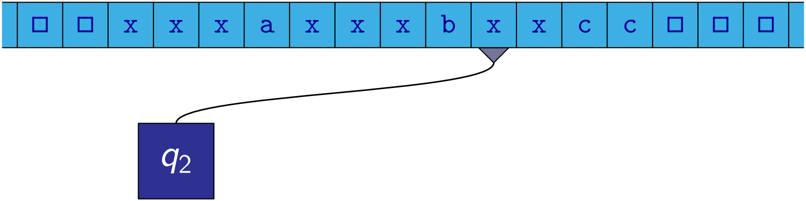
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



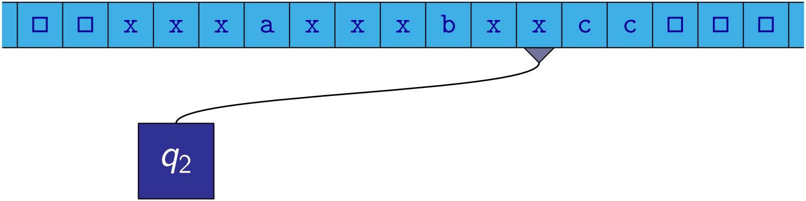
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



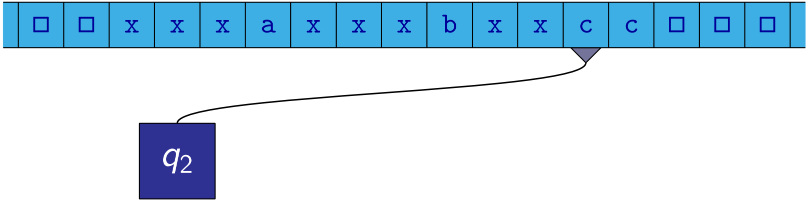
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



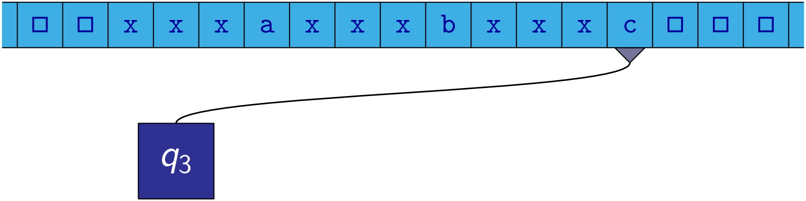
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



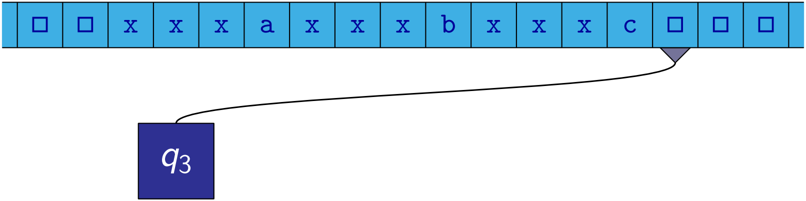
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



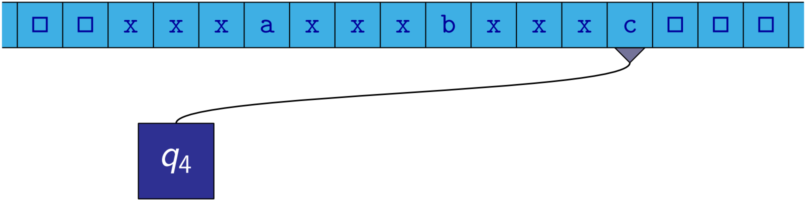
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



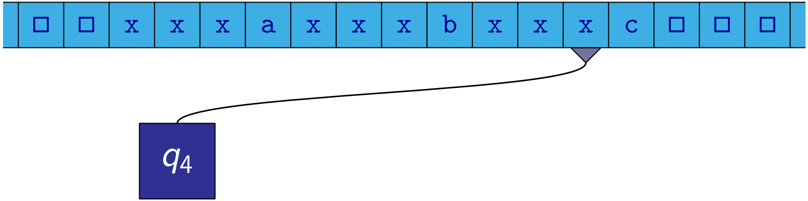
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\Box, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\Box$	a	b	c	x
q_0	$(q_{acc}, \Box, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \Box, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \Box, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \Box, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \Box, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



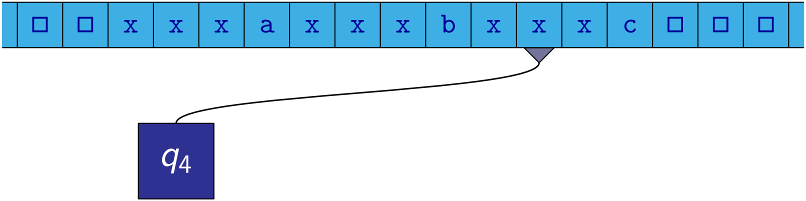
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



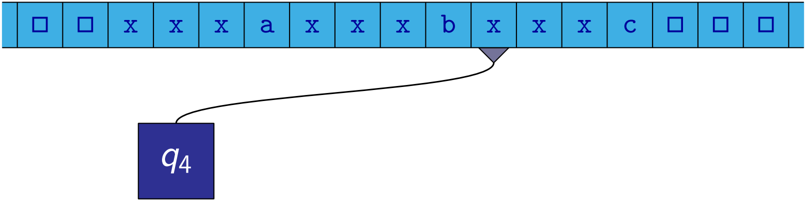
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



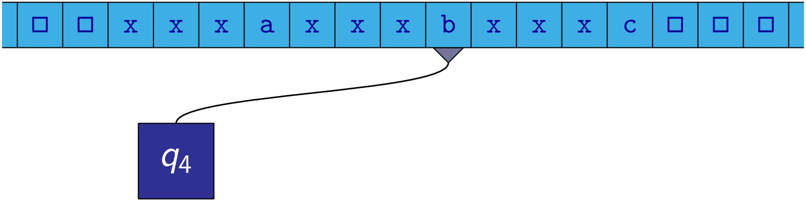
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$

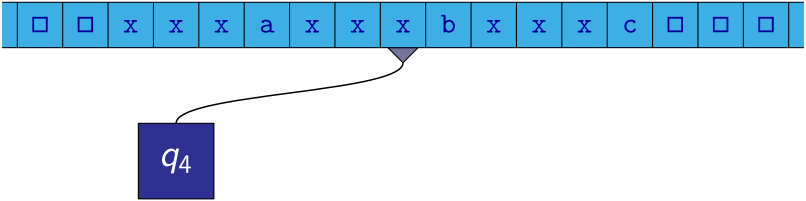


Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



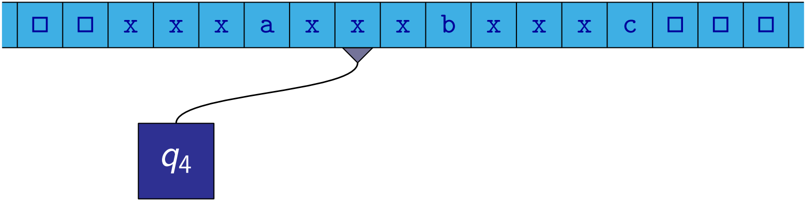
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



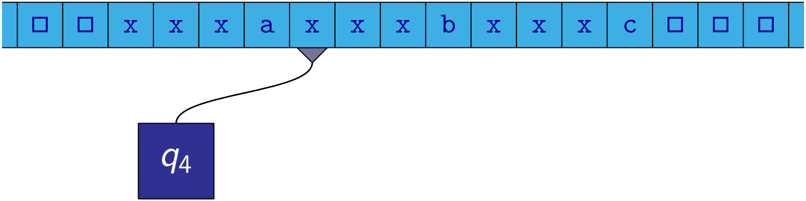
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



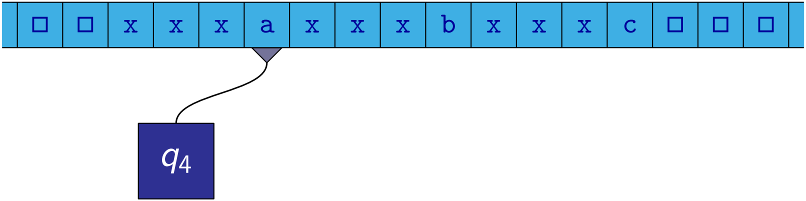
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



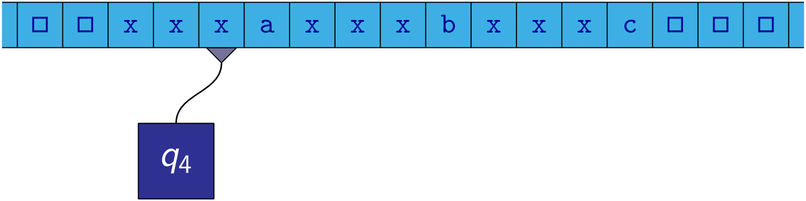
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



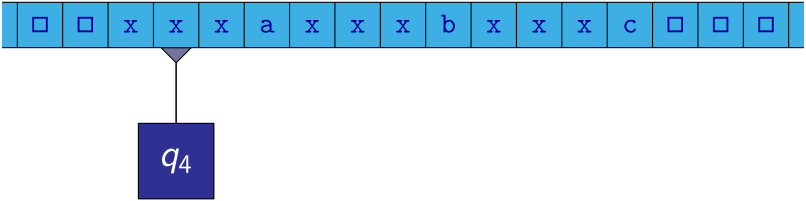
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



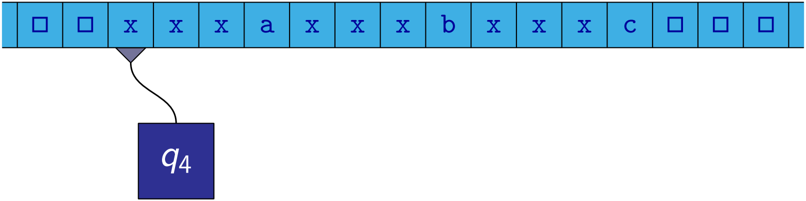
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



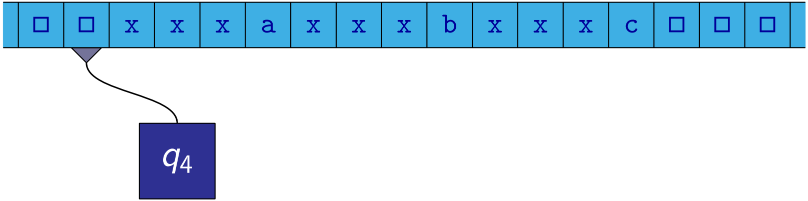
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



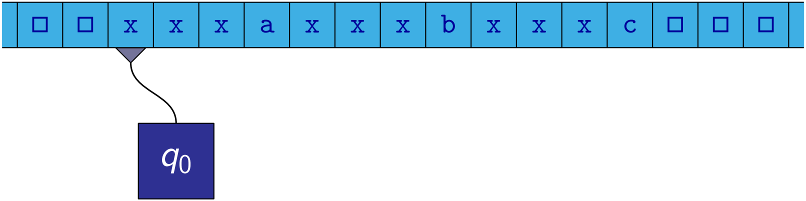
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



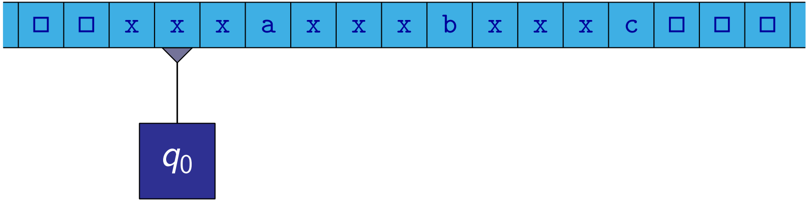
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



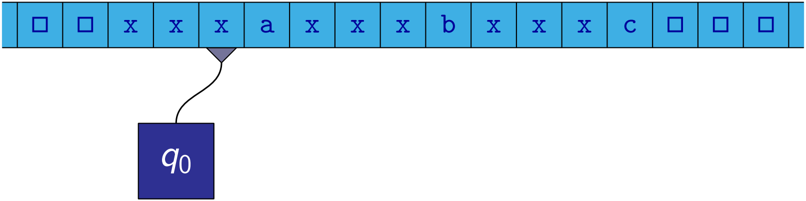
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\Box, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\Box$	a	b	c	x
q_0	$(q_{acc}, \Box, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \Box, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \Box, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \Box, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \Box, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



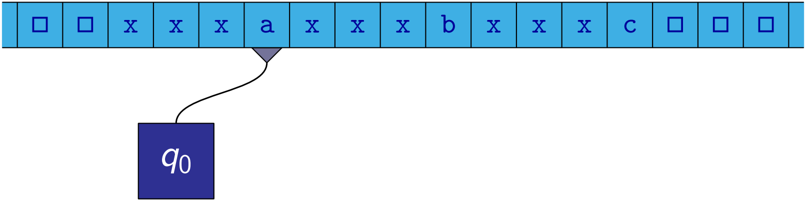
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\Box, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\Box$	a	b	c	x
q_0	$(q_{acc}, \Box, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \Box, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \Box, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \Box, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \Box, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



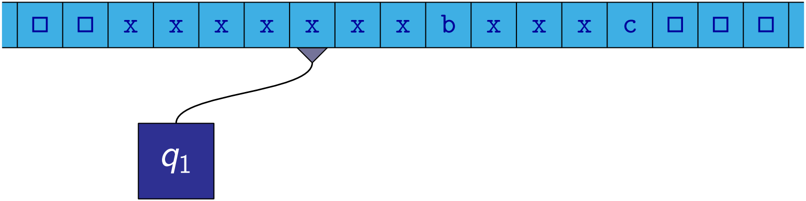
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\Box, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\Box$	a	b	c	x
q_0	$(q_{acc}, \Box, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \Box, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \Box, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \Box, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \Box, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



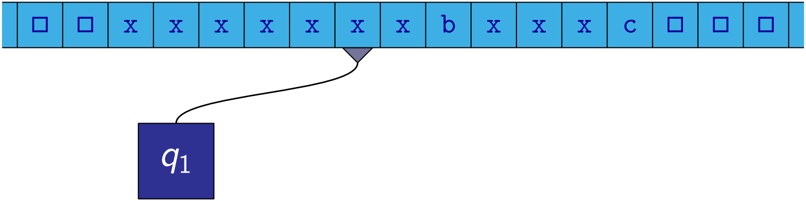
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



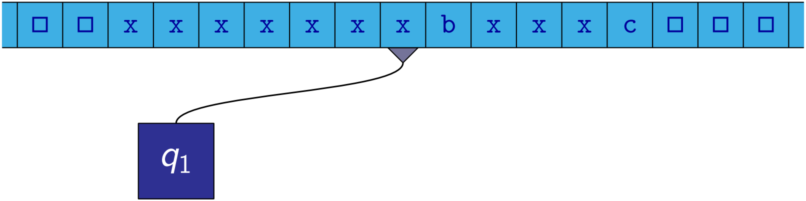
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad$ $\Gamma = \{\Box, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\Box$	a	b	c	x
q_0	$(q_{acc}, \Box, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \Box, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \Box, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \Box, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \Box, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



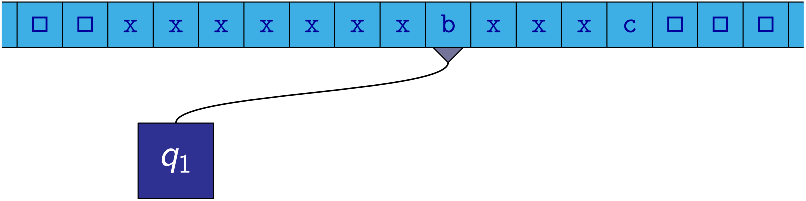
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



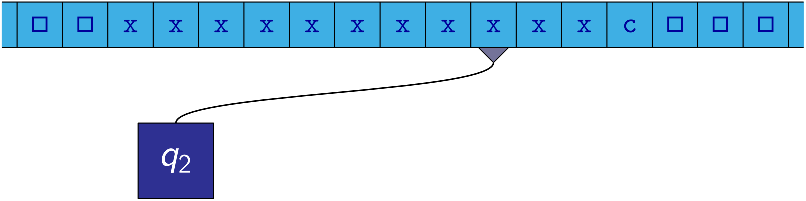
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	$\mathtt{a}$	$\mathtt{b}$	$\mathtt{c}$	$\mathtt{x}$
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



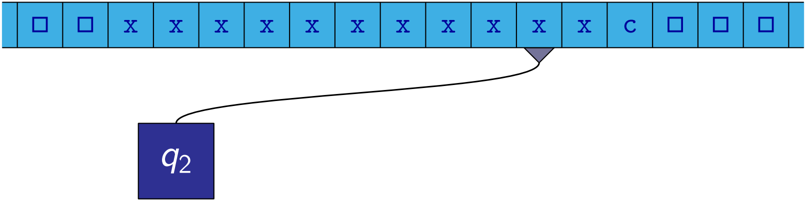
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



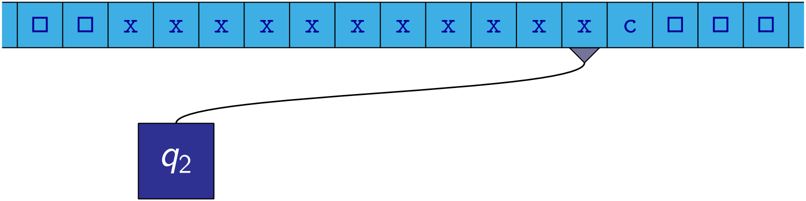
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



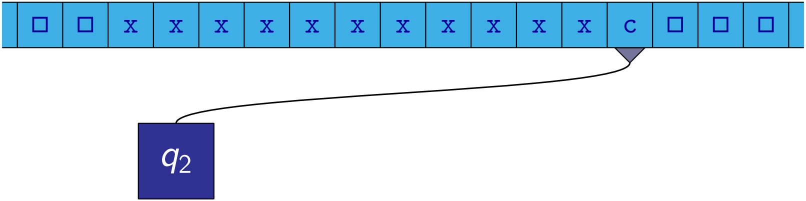
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



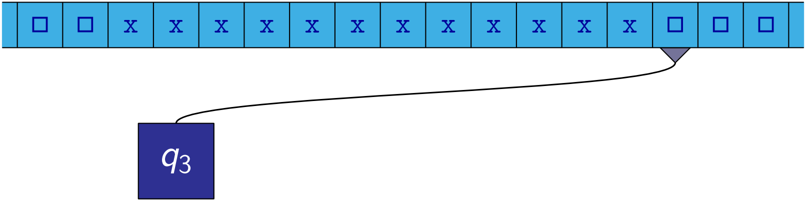
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



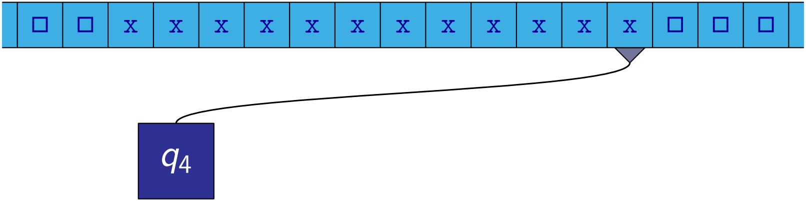
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$

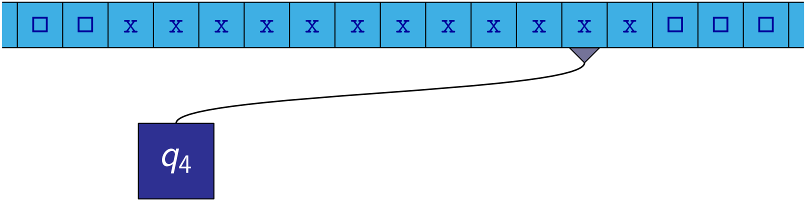


Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$

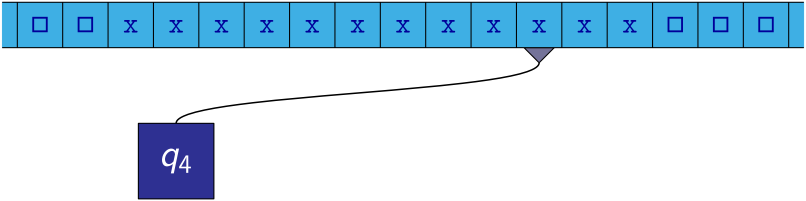


Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



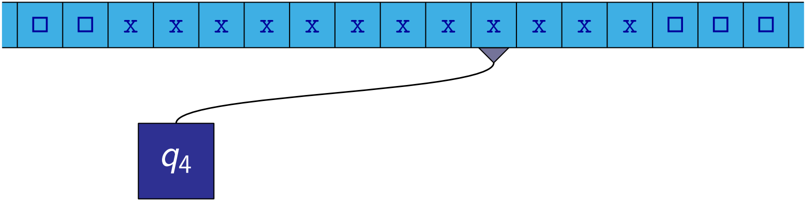
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



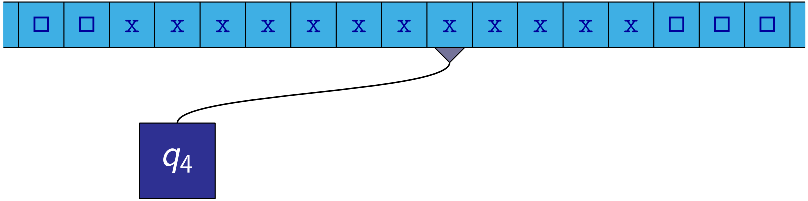
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$

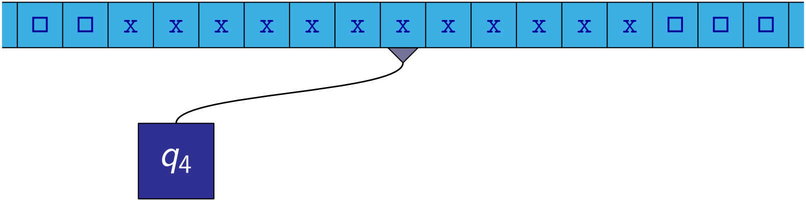


Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad$ $\Gamma = \{\Box, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\Box$	a	b	c	x
q_0	$(q_{acc}, \Box, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \Box, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \Box, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \Box, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \Box, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$

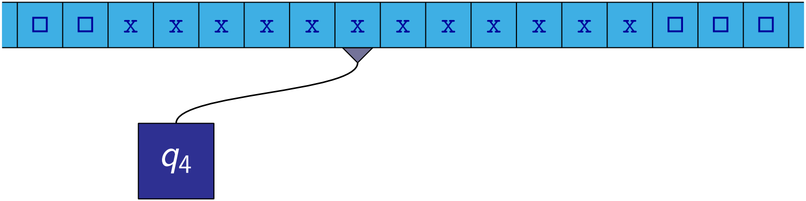


Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	$\mathtt{a}$	$\mathtt{b}$	$\mathtt{c}$	$\mathtt{x}$
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



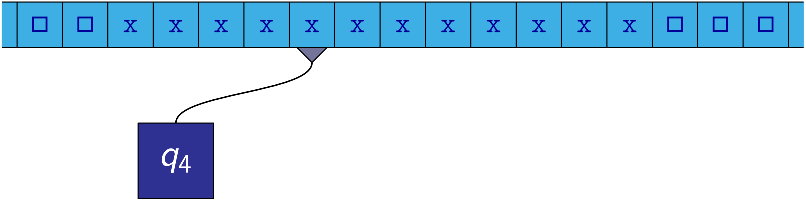
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\Box, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\Box$	a	b	c	x
q_0	$(q_{acc}, \Box, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \Box, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \Box, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \Box, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \Box, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



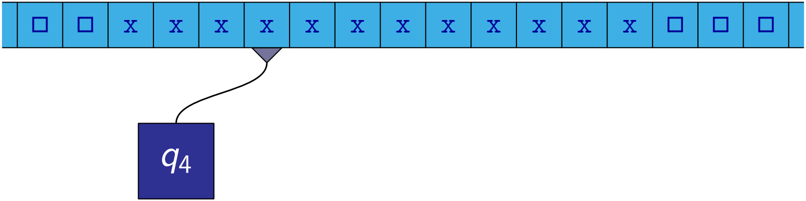
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



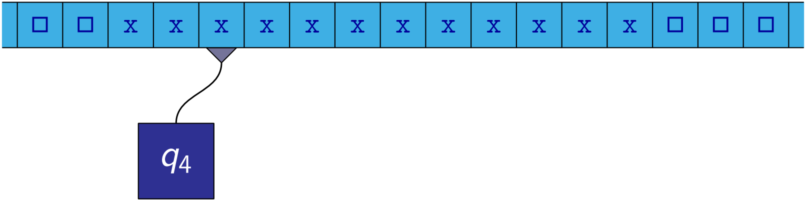
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



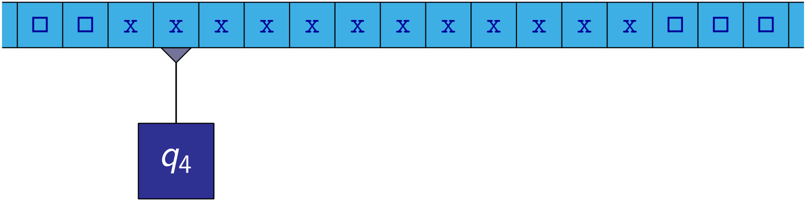
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



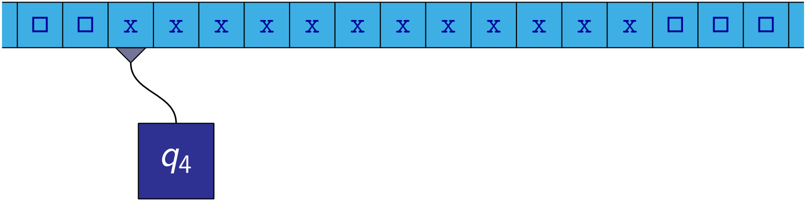
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



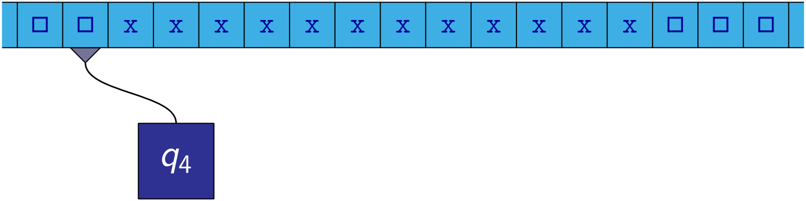
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



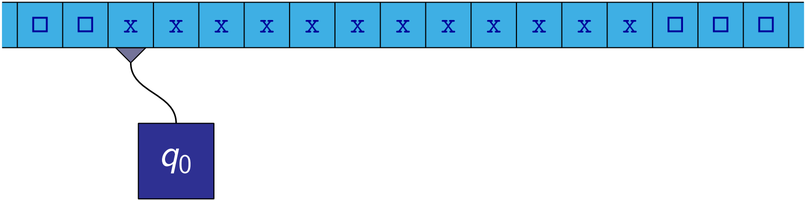
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



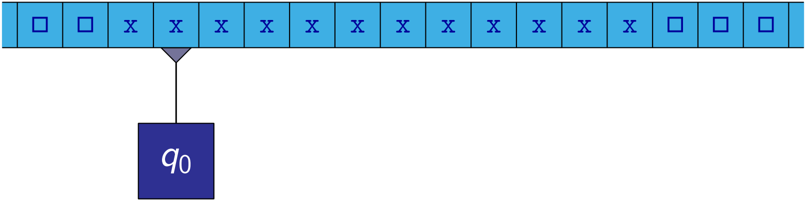
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



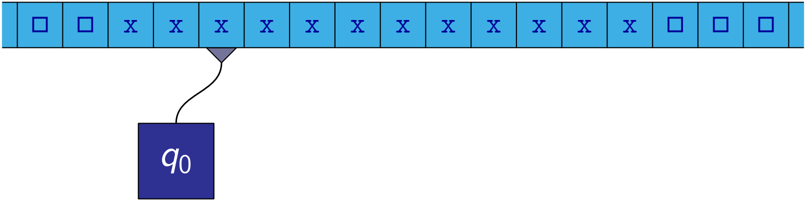
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



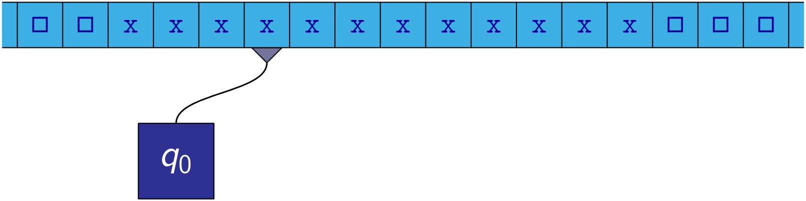
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



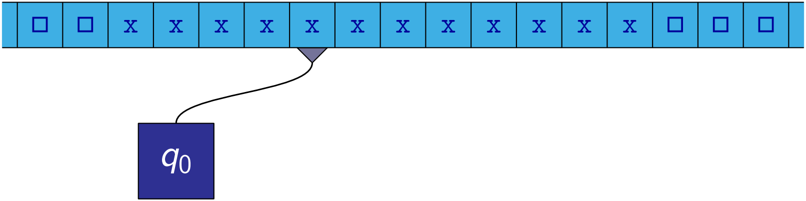
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



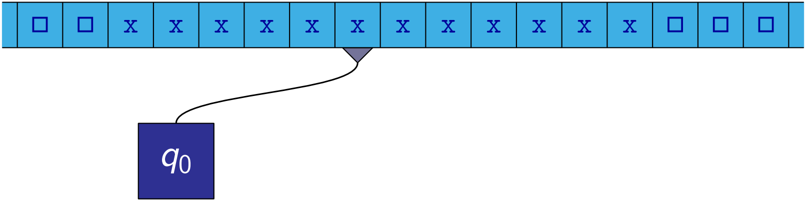
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



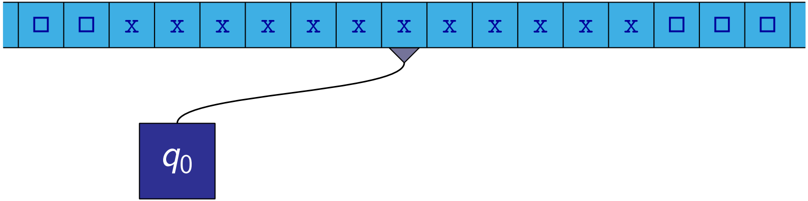
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



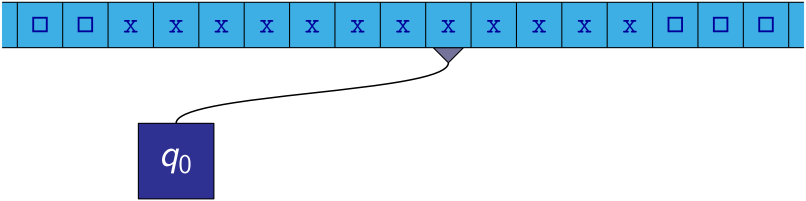
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



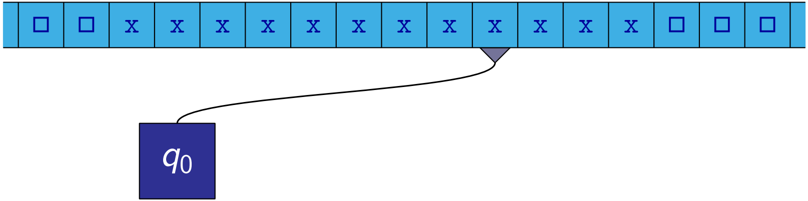
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad$ $\Gamma = \{\Box, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\Box$	a	b	c	x
q_0	$(q_{acc}, \Box, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \Box, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \Box, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \Box, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \Box, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



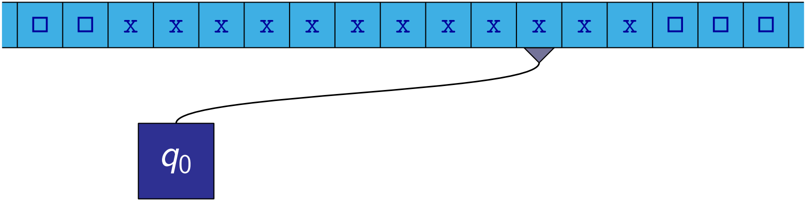
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



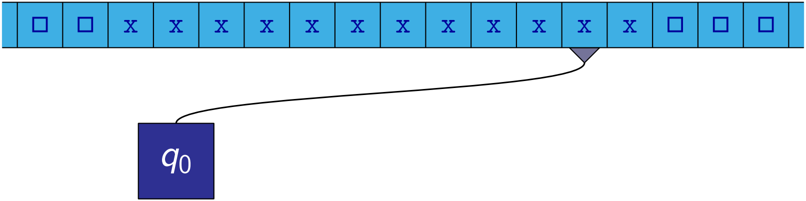
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



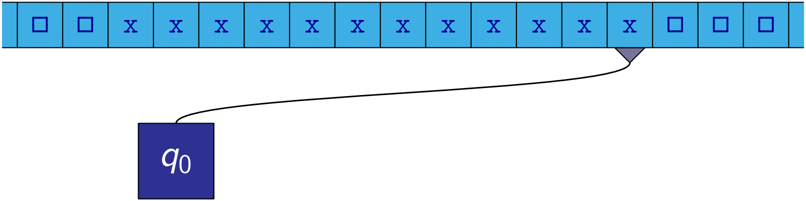
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\quad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\quad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$

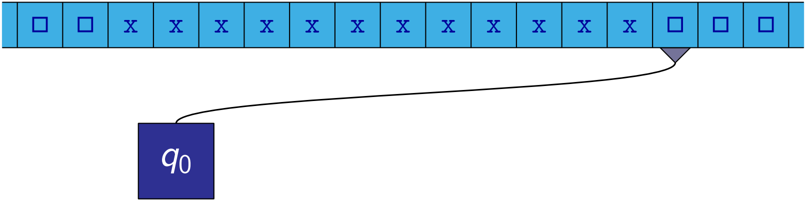


Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



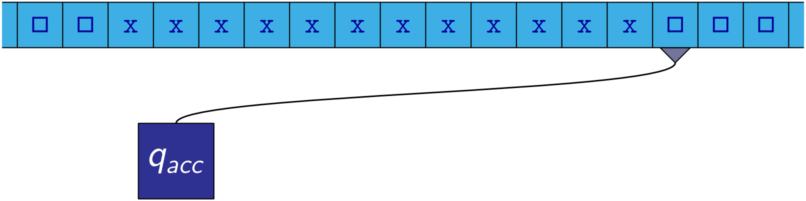
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

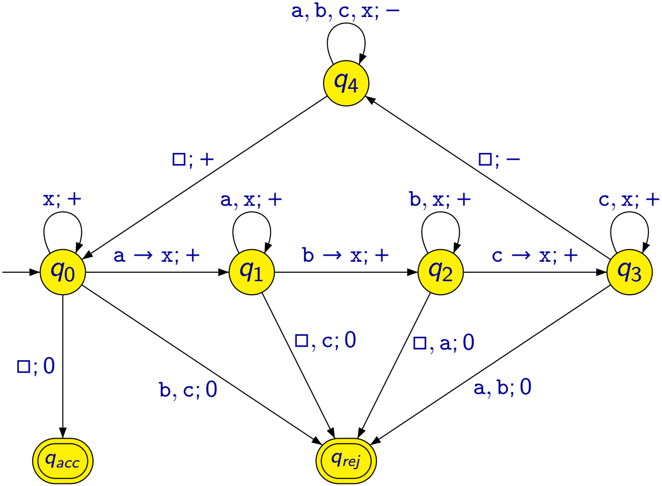
$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $\qquad F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ $\qquad \Gamma = \{\square, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{x}\}$

δ	$\square$	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_0, \mathtt{x}, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, \mathtt{a}, +1)$	$(q_2, \mathtt{x}, +1)$	$(q_{rej}, \mathtt{c}, 0)$	$(q_1, \mathtt{x}, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_2, \mathtt{b}, +1)$	$(q_3, \mathtt{x}, +1)$	$(q_2, \mathtt{x}, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, \mathtt{a}, 0)$	$(q_{rej}, \mathtt{b}, 0)$	$(q_3, \mathtt{c}, +1)$	$(q_3, \mathtt{x}, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, \mathtt{a}, -1)$	$(q_4, \mathtt{b}, -1)$	$(q_4, \mathtt{c}, -1)$	$(q_4, \mathtt{x}, -1)$



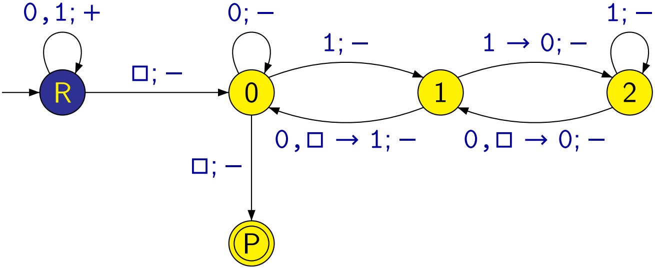
Turing Machine

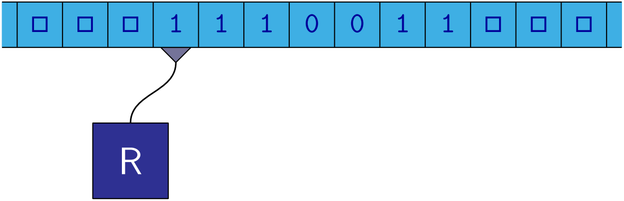


Turing Machine

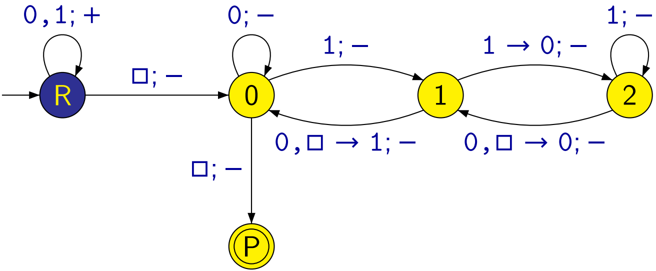
- A Turing machine can give not only answers $\textsc{Yes}$ or $\textsc{No}$ but it can also compute a function that assigns to each word from Σ^* some other word (from Γ^*).
- A word assigned to a word w is the word that remains on the tape after the computation over the word w when we remove all symbols $\square$.

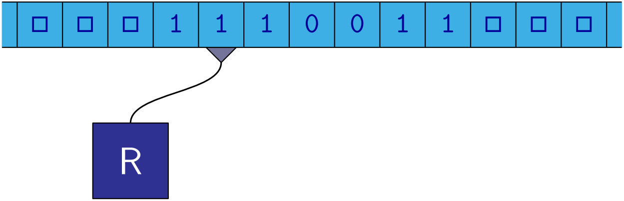
Turing Machine – Multiplication by Three



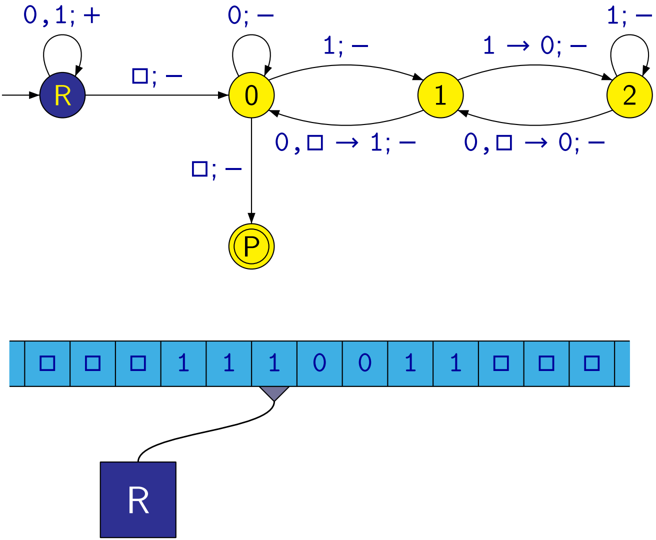


Turing Machine – Multiplication by Three

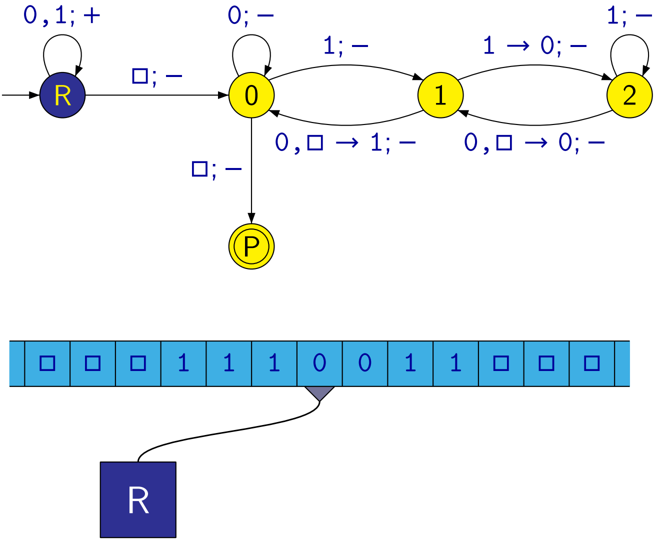




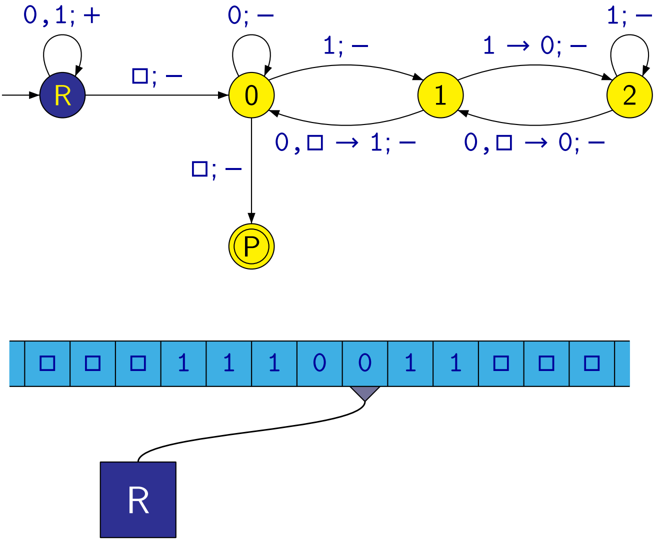
Turing Machine – Multiplication by Three



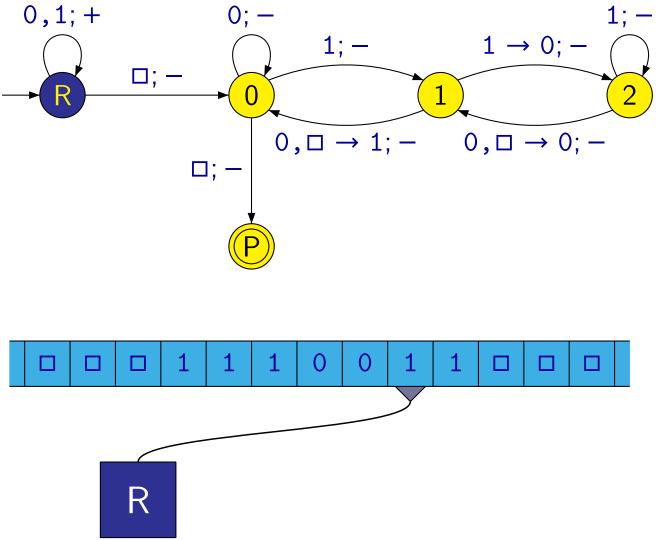
Turing Machine – Multiplication by Three



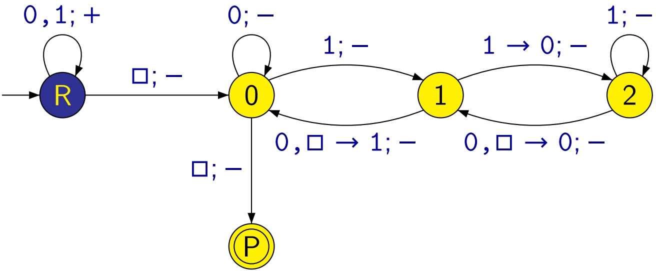
Turing Machine – Multiplication by Three

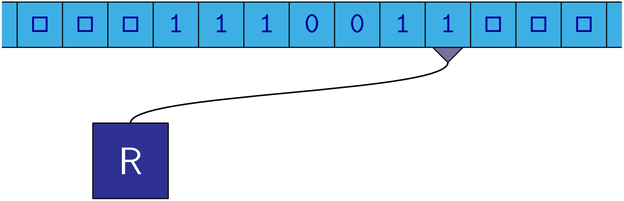


Turing Machine – Multiplication by Three

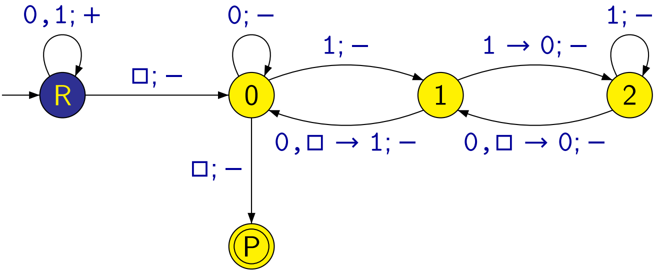


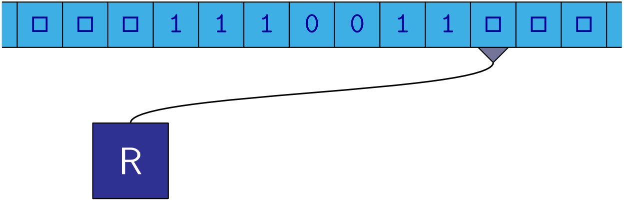
Turing Machine – Multiplication by Three



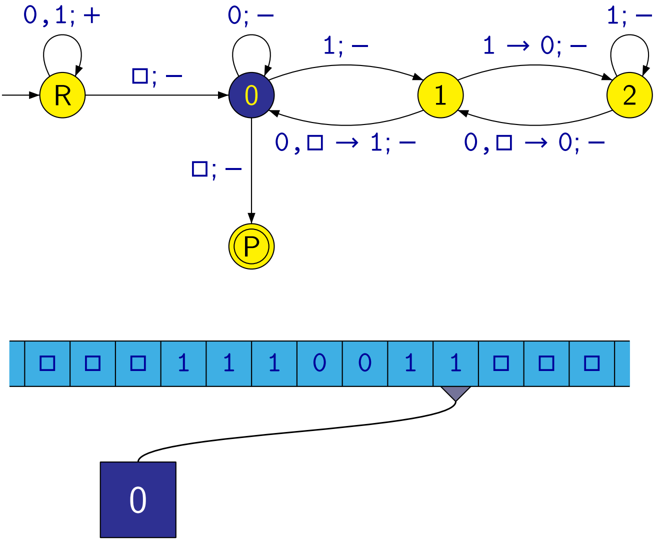


Turing Machine – Multiplication by Three

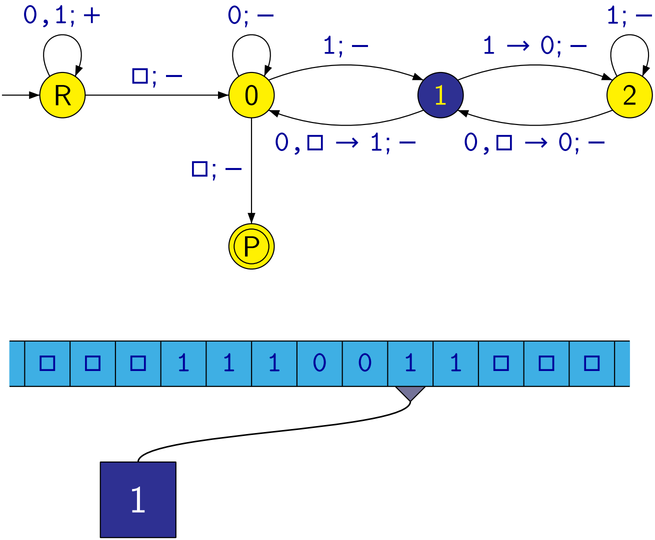




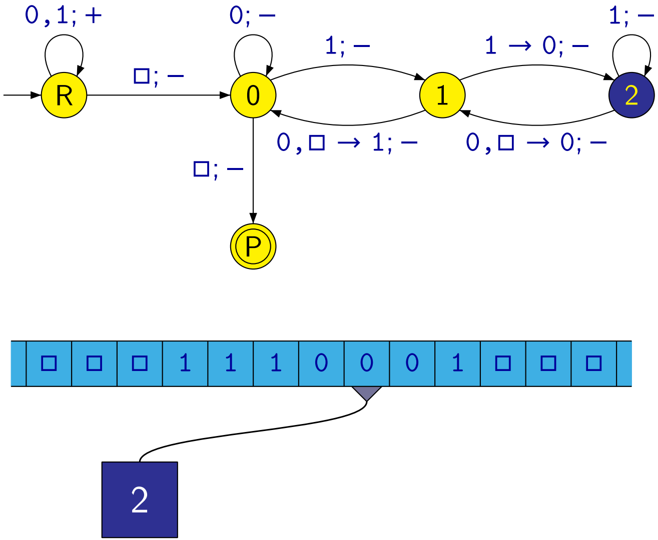
Turing Machine – Multiplication by Three



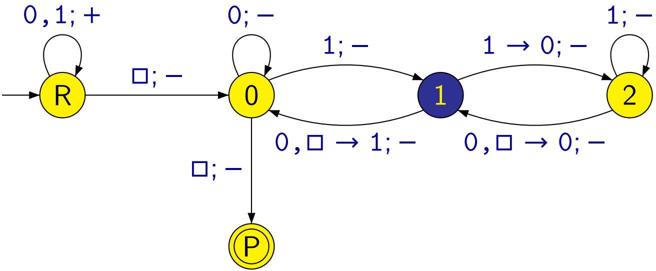
Turing Machine – Multiplication by Three

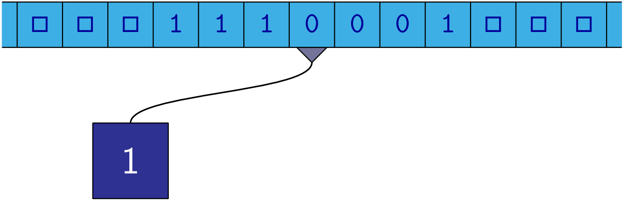


Turing Machine – Multiplication by Three

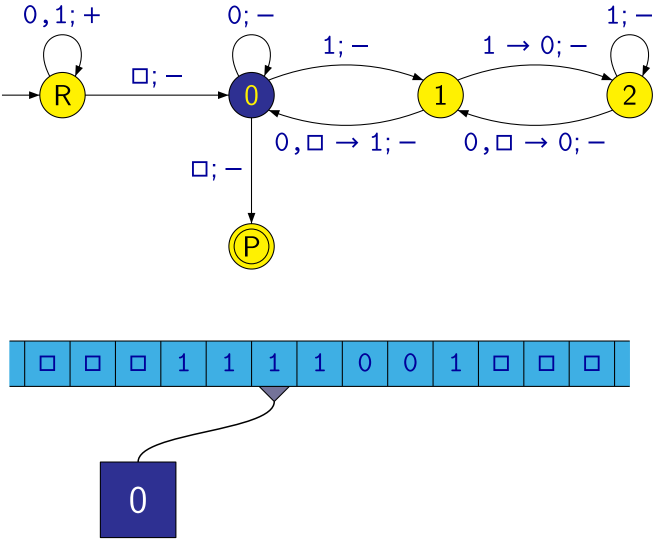


Turing Machine – Multiplication by Three

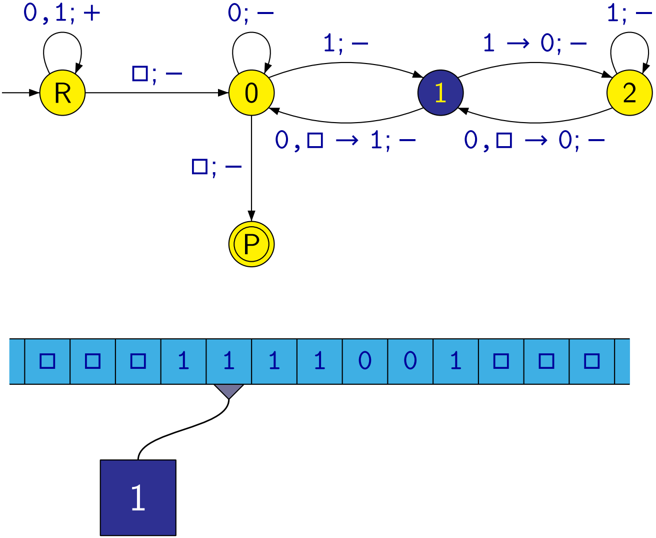




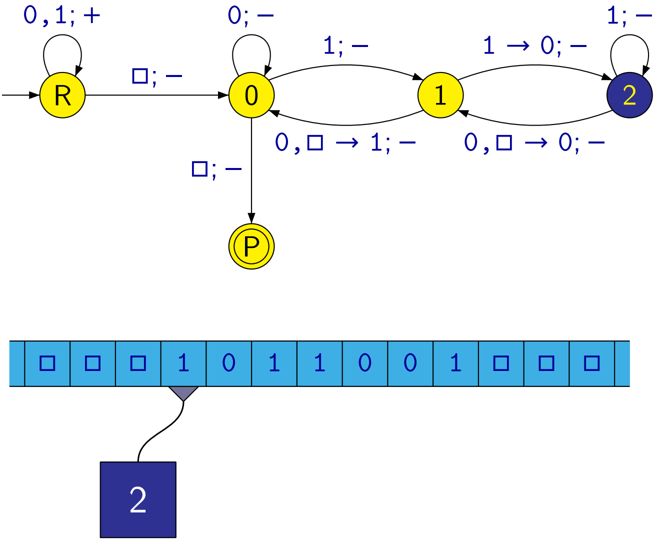
Turing Machine – Multiplication by Three



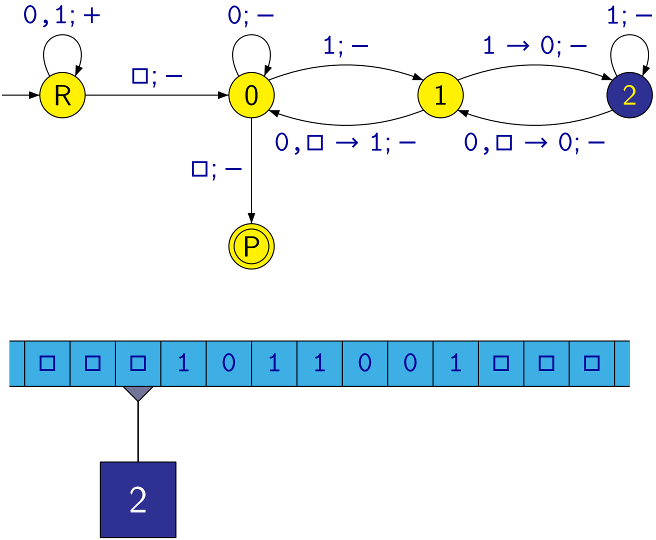
Turing Machine – Multiplication by Three



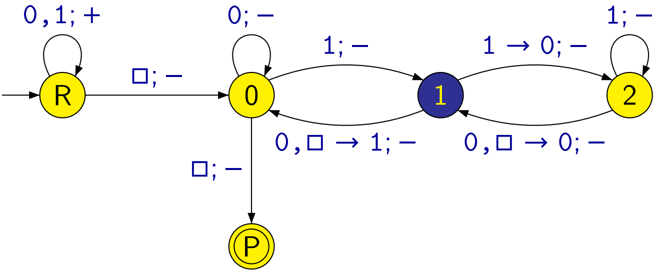
Turing Machine – Multiplication by Three

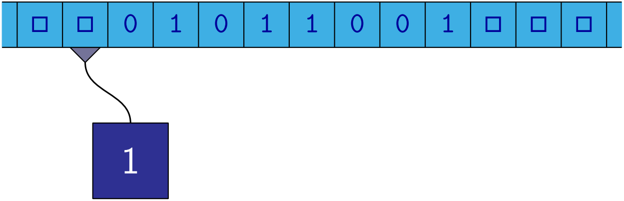


Turing Machine – Multiplication by Three

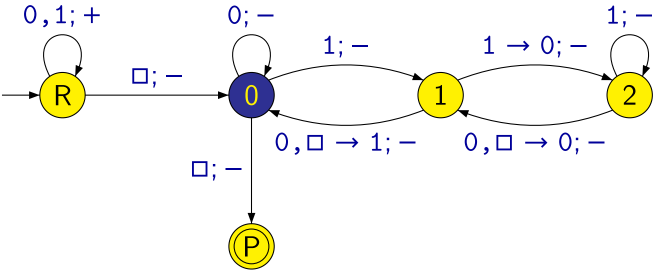


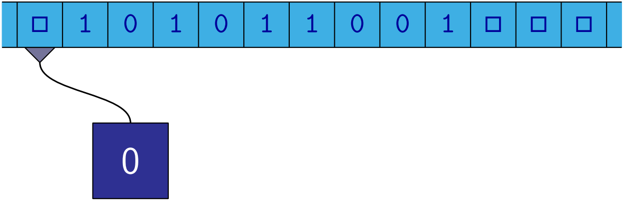
Turing Machine – Multiplication by Three



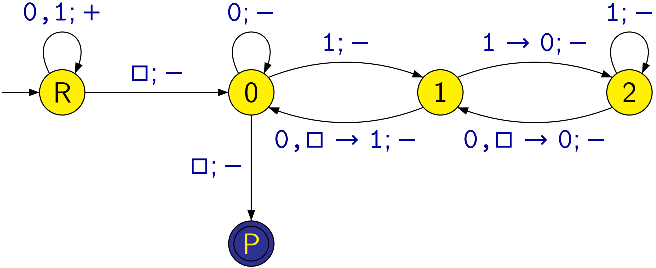


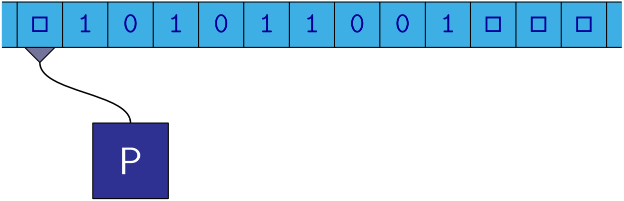
Turing Machine – Multiplication by Three





Turing Machine – Multiplication by Three





Nondeterministic Turing Machines

We can also consider **nondeterministic Turing machines** where for every state q and symbol b the transition function $\delta(q, b)$ specifies several different triples (q', b', d).

The machine can choose any of them.

The machine accepts a word w iff it has at least one computation where w is accepted.

Remark: For every nondeterministic Turing machine, there can be constructed an equivalent deterministic Turing machine.

Nondeterministic Turing Machines

Formally, the only difference in the definition of a deterministic and a nondeterministic Turing machine $\mathcal{M} = (Q, \Sigma, \Gamma, \delta, q_0, F)$ is the definition of the transition function δ:

- **Deterministic** Turing machine:

$$\delta : (Q - F) \times \Gamma \rightarrow Q \times \Gamma \times \{-1, 0, +1\}$$

- **Nondeterministic Turing machine**:

$$\delta : (Q - F) \times \Gamma \rightarrow \mathcal{P}(Q \times \Gamma \times \{-1, 0, +1\})$$

Remark: For nondeterministic Turing machines, it makes a little sense to consider other set of final state than $F = \{q_{acc}, q_{rej}\}$.

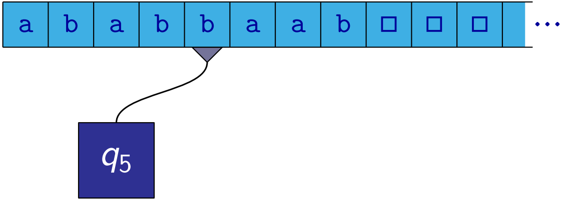
Variants of Turing Machines

- The definition of Turing machine given before is just one of many variants.
- Here we give several examples of differences between different variants of Turing machines.
- Almost all these variants of Turing machines are able to accept or recognize the same languages and to compute the same functions.
- There can be (but need not be) big differences between variants with respect to their running time and an amount of used memory.
- All these variants can be considered in a deterministic and a nondeterministic version.

Variants of Turing Machines

One-sided or **two-sided** infinite tape:

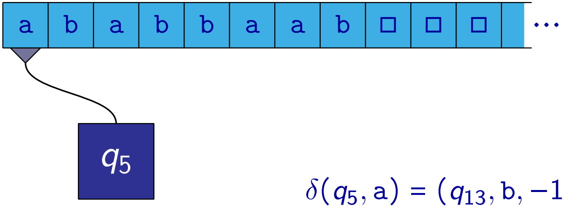
- In the previous definition, we have considered a tape that is infinite in both directions — to the left and to the right.
- Instead, it is sometimes considered a tape that is infinite only to the right.



It is necessary to define what should happen when the head is on the leftmost cell of the tape and, according to the transition function, it should move to the left.

Two most common possibilities:

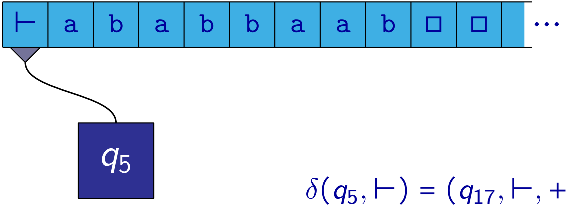
- An "error" occurs and the computation is (unsuccesfully) ended:



$$\delta(q_5, \mathtt{a}) = (q_{13}, \mathtt{b}, -1)$$

- The left end of the tape contains a "marker" represented by a special symbol $\vdash\,\in (\Gamma - \Sigma)$.

  This marker can not be overwritten and a move to the left is forbidden on this symbol, i.e., for each $q \in Q$ it holds that if $\delta(q, \vdash) = (q', b, d)$ then $b = \vdash$ a $d \in \{0, +1\}$.



$\delta(q_5, \vdash) = (q_{17}, \vdash, +1)$

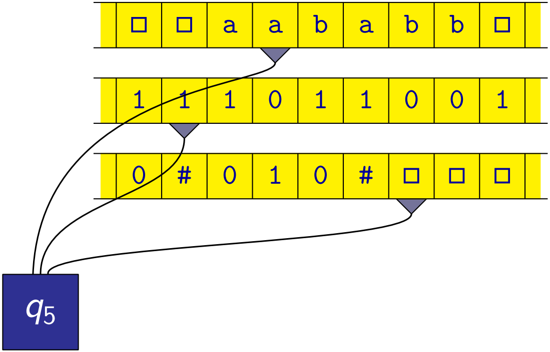
Variants of Turing Machines

Remark: The possibility that a computation can end unsuccessfully because of an error where it is no possible to continue from the given configuration is quite common also for other types of machines we will consider.

Generally, the following possibilities can happen in a computation:

- The computation ends successfully in a final configuration that corresponds to a correct halting.
- The computation is stuck in a configuration that is not final but it is not possible to continue there — this is considered as a computation ending with an error.
- The computation never halts.

Multitape Turing machines are often considered.



Variants of Turing Machines

In the case of a multitape machines:

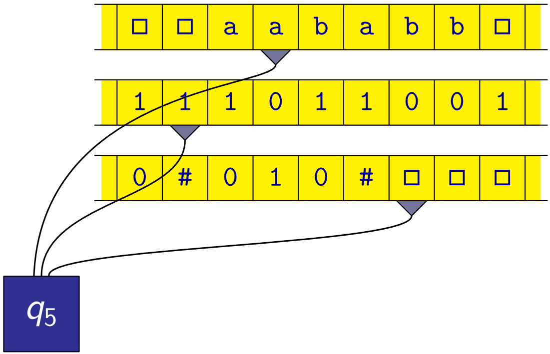
- Each of k tapes has its own alphabet, i.e., we have tape alphabets Γ_1, Γ_2, ..., Γ_k.
- The transition function δ is of the type

$$(Q - F) \times \Gamma_1 \times \cdots \times \Gamma_k \rightarrow Q \times \Gamma_1 \times \{-1, 0, +1\} \times \cdots \times \Gamma_k \times \{-1, 0, +1\}$$

Example:

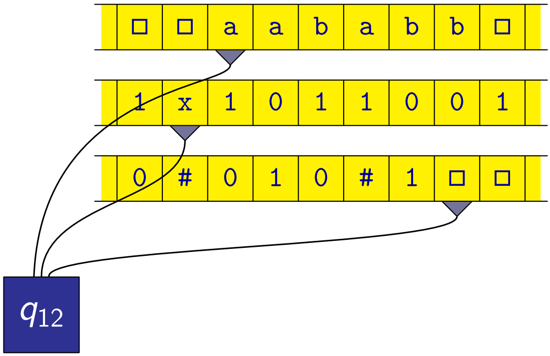
$$\delta(q_5, \mathtt{a}, 1, \Box) = (q_{12}, \mathtt{a}, -1, \mathtt{x}, 0, 1, +1)$$

Example:



$$\delta(q_5, \mathtt{a}, 1, \square) = (q_{12}, \mathtt{a}, -1, \mathtt{x}, 0, 1, +1)$$

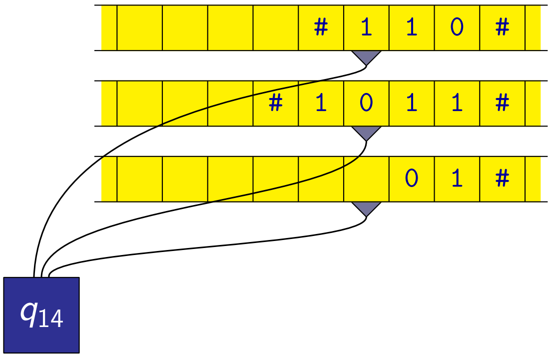
Example:



$$\delta(q_5, \mathtt{a}, 1, \Box) = (q_{12}, \mathtt{a}, -1, \mathtt{x}, 0, 1, +1)$$

Variants of Turing Machines

Example: A machine that gets as an input two natural numbers written in binary and separated by symbols # (e.g., number 6 and 11 will be written as "#110#" a "#1011#").

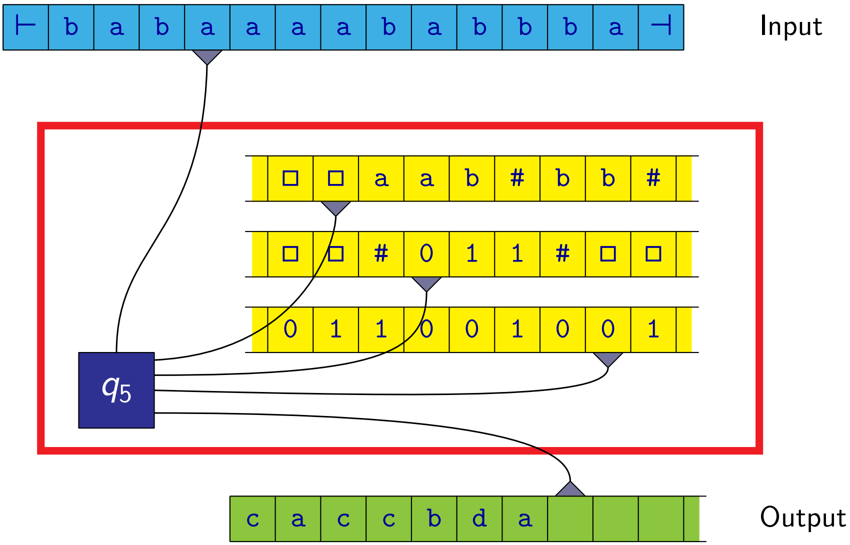


Variants of Turing Machines

Multitape machines often use one of its tapes as an input tape and one of its tapes as an output tape. Other tapes are used as working tapes:

- **Input tape** — it contains an input word, the machine can not write on it (it is read-only), it is not infinite
- **Working tapes** — the machine can read from them and write on them (they are read/write), at the beginning of a computation they are empty (they contain only symbols $\square$)
- **Output tape** — the machine can only write on it (it is write-only), it can not read from it, it is empty at the beginning of a computation, the head can move only from the left to the right

Variants of Turing Machines



Variants of Turing Machines

If a machine has a special separate input tape (which is read-only), the following two variants are typically used:

- The head on this tape can move to the left and to the right.

  In this case, an input word $w \in \Sigma^*$ is bounded from the left and from the right using "endmarkers", i.e., special symbols $\vdash, \dashv \in (\Gamma - \Sigma)$.

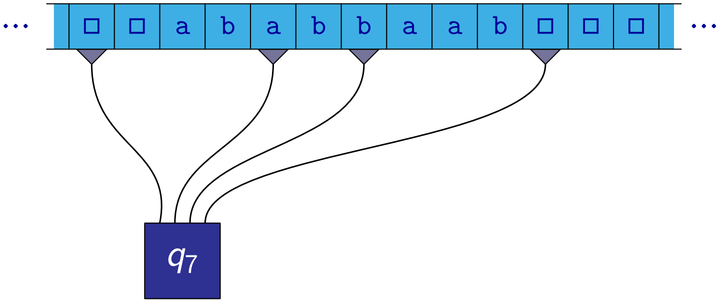
- The head can move only from left to right.

Remark: The variant with possible movement in both directions and endmarkers is more common.

If it is not specified otherwise, we will consider this variant.

Variants of Turing Machines

Instead of several tapes, we can consider **several heads** on one tape:



Variants of Turing Machines

In the variant with several heads on one tape, it is necessary to specify:

- If there can be more than one head in the same time on one tape cell.
- If this is the case, what is the behaviour of the machine if several heads occurring on the same cell want to write different symbols on this cell.
- Whether the given machine can detect the situation when several head are on the same cell.

Remark: Of course, in general we can consider machines with several tapes where each of these tapes is equipped with several heads.

Variants of Turing Machines

Consider a machine with several tapes and with arbitrary number of heads on each tape.

Instead of describing a transition function that works with all heads in each step, we can alternatively describe the behaviour of the machine by a **program** consisting of simpler instructions of the following types:

- to move a given head by one cell to the left
- to move a given head by one cell to the right
- to write a specified symbol on the given position of a specified head on a tape
- to read one symbol from a position of a given head and to branch the program according this symbol (i.e., to go to different states of the control unit)

Variants of Turing Machines

So far we considered only **linear** (one-dimensitional) tapes.

Instead, the memory with cells (where every cell contains one symbol from some alphabet) can have some other structure.

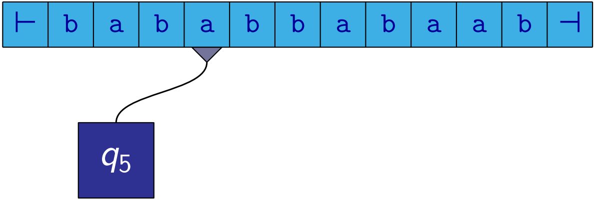
For example:

- two-dimensional **square grid**
  — a movement of a head into four directions: left, right, up, down
- d-dimensional memory for some $d = 3, 4, \ldots$
  (three-dimensional, four-dimensional, etc.)
- a memory organized in a form of an (infinite) tree
- ...

Linear Bounded Automaton

Linear bounded automaton (**LBA**):

- A nondeterministic Turing machine that can use only the part of the tape where its input word is written.
- Cells of the tape, which at the beginning contain symbols of an input word, can be arbitrarily overwritten during a computation.
- Left and right endmarkers around the word. These endmarkers can not be overwritten.
- It is not possible to move the head to the left of the left endmarker and to the right of the right endmarker.



Linear Bounded Automaton

- Linear bounded automata can be considered in both **deterministic** and **nondeterministic** version.
- The nondeterministic version is considered as the default (i.e., if it is not specified otherwise).
- The question whether every language that can be recognized by a nondeterministic LBA can be also also recognized by a deterministic LBA is an open problem.

Remark: From the point of view of languages that they are able to accept or recognize and from the point view of functions that they can compute, linear bounded automata are considerably weaker than Turing machines that can use memory of unbounded size (in the form of an infinite tape).

Chomsky Hierarchy

Generative Grammars

Definition

A **generative grammar** is a tuple $\mathcal{G} = (\Pi, \Sigma, S, P)$, where

- Π is a finite set of nonterminals
- Σ is a finite set of terminals, $\Pi \cap \Sigma = \emptyset$
- $S \in \Pi$ is the initial nonterminal
- P is a finite set of rules of the form $\alpha \to \beta$, where $\alpha \in (\Pi \cup \Sigma)^* \Pi (\Pi \cup \Sigma)^*$ and $\beta \in (\Pi \cup \Sigma)^*$.

Example of a rule:

$$CaECb \to bDFbBDaC$$

Remark: This type of grammar is also called **type-0** grammars, **unrestricted** grammars, or **phrase structure grammars**.

Generative Grammars

Let us assume that we have a generative grammar $\mathcal{G} = (\Pi, \Sigma, S, P)$.

Relation $\Rightarrow \subseteq (\Pi \cup \Sigma)^* \times (\Pi \cup \Sigma)^*$:

- $\mu_1 \alpha \mu_2 \;\Rightarrow\; \mu_1 \beta \mu_2$ if $\alpha \rightarrow \beta$ is a rule from P

Example: If $(BcE \rightarrow DDaBb) \in P$ then

$$CaBC\,BcE\,AccABb \;\Rightarrow\; CaBC\,DDaBb\,AccABb$$

A **language** $\mathcal{L}(\mathcal{G})$ generated by a grammar $\mathcal{G} = (\Pi, \Sigma, S, P)$ is the set of all words over alphabet Σ that can be derived by some derivation from the initial nonterminal S using rules from P, i.e.,

$$\mathcal{L}(\mathcal{G}) = \{w \in \Sigma^* \mid S \Rightarrow^* w\}$$

Generative Grammars

Let us assume that we have a generative grammar $\mathcal{G} = (\Pi, \Sigma, S, P)$.

Relation $\Rightarrow \subseteq (\Pi \cup \Sigma)^* \times (\Pi \cup \Sigma)^*$:

- $\mu_1 \alpha \mu_2 \ \Rightarrow \ \mu_1 \beta \mu_2$ if $\alpha \ \rightarrow \ \beta$ is a rule from P

Example: If $(BcE \ \rightarrow \ DDaBb) \in P$ then

$$CaBC\underline{BcE}AccABb \ \Rightarrow \ CaBC\underline{DDaBb}AccABb$$

A **language** $\mathcal{L}(\mathcal{G})$ generated by a grammar $\mathcal{G} = (\Pi, \Sigma, S, P)$ is the set of all words over alphabet Σ that can be derived by some derivation from the initial nonterminal S using rules from P, i.e.,

$$\mathcal{L}(\mathcal{G}) = \{w \in \Sigma^* \mid S \Rightarrow^* w\}$$

Generative Grammars

Example: A grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abc \\ cQ &\rightarrow Qc \\ bQc &\rightarrow bbcc \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

S

Generative Grammars

Example: A grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\to aSQ \\ S &\to abc \\ cQ &\to Qc \\ bQc &\to bbcc \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$S \Rightarrow aSQ$$

Generative Grammars

Example: A grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abc \\ cQ &\rightarrow Qc \\ bQc &\rightarrow bbcc \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\begin{aligned} S &\Rightarrow aSQ \\ &\Rightarrow aaSQQ \end{aligned}$$

Example: A grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abc \\ cQ &\rightarrow Qc \\ bQc &\rightarrow bbcc \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\begin{aligned} S &\Rightarrow aSQ \\ &\Rightarrow aaSQQ \\ &\Rightarrow aaaSQQQ \end{aligned}$$

Generative Grammars

Example: A grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abc \\ cQ &\rightarrow Qc \\ bQc &\rightarrow bbcc \end{aligned}$$

A derivation of word $aaaabbbbcccc$:

$$\begin{aligned} S &\Rightarrow aSQ \\ &\Rightarrow aaSQQ \\ &\Rightarrow aaaSQQQ \\ &\Rightarrow aaaaSQQQQ \end{aligned}$$

Generative Grammars

Example: A grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{aligned}
S &\rightarrow aSQ \\
S &\rightarrow abc \\
cQ &\rightarrow Qc \\
bQc &\rightarrow bbcc
\end{aligned}
$$

A derivation of word $aaaaabbbbbccccc$:

$$
\begin{aligned}
S &\Rightarrow aSQ \\
&\Rightarrow aaSQQ \\
&\Rightarrow aaaSQQQ \\
&\Rightarrow aaaaSQQQQ \\
&\Rightarrow aaaaabcQQQQ
\end{aligned}
$$

Generative Grammars

Example: A grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abc \\ cQ &\rightarrow Qc \\ bQc &\rightarrow bbcc \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\begin{aligned} S &\Rightarrow aSQ \\ &\Rightarrow aaSQQ \\ &\Rightarrow aaaSQQQ \\ &\Rightarrow aaaaSQQQQ \\ &\Rightarrow aaaaabcQQQQ \\ &\Rightarrow aaaaabQcQQQ \end{aligned}$$

Generative Grammars

Example: A grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{aligned}
S &\rightarrow aSQ \\
S &\rightarrow abc \\
cQ &\rightarrow Qc \\
bQc &\rightarrow bbcc
\end{aligned}
$$

A derivation of word $aaaaabbbbbccccc$:

$$
\begin{aligned}
S &\Rightarrow aSQ \\
&\Rightarrow aaSQQ \\
&\Rightarrow aaaSQQQ \\
&\Rightarrow aaaaSQQQQ \\
&\Rightarrow aaaaabcQQQQ \\
&\Rightarrow aaaaabQcQQQ \\
&\Rightarrow aaaaabbccQQQ
\end{aligned}
$$

Generative Grammars

Example: A grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{aligned}
S &\rightarrow aSQ \\
S &\rightarrow abc \\
cQ &\rightarrow Qc \\
bQc &\rightarrow bbcc
\end{aligned}
$$

A derivation of word $aaaaabbbbbccccc$:

$$
\begin{aligned}
S &\Rightarrow aSQ \\
&\Rightarrow aaSQQ \\
&\Rightarrow aaaSQQQ \\
&\Rightarrow aaaaSQQQQ \\
&\Rightarrow aaaaabcQQQQ \\
&\Rightarrow aaaaabQcQQQ \\
&\Rightarrow aaaaabbccQQQ \\
&\Rightarrow aaaaabbcQcQQ
\end{aligned}
$$

Generative Grammars

Example: A grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{aligned}
S &\to aSQ \\
S &\to abc \\
cQ &\to Qc \\
bQc &\to bbcc
\end{aligned}
$$

A derivation of word $aaaaabbbbbccccc$:

$$
\begin{aligned}
S &\Rightarrow aSQ \\
&\Rightarrow aaSQQ \\
&\Rightarrow aaaSQQQ \\
&\Rightarrow aaaaSQQQQ \\
&\Rightarrow aaaaabcQQQQ \\
&\Rightarrow aaaaabQcQQQ \\
&\Rightarrow aaaaabbccQQQ \\
&\Rightarrow aaaaabbcQcQQ
\end{aligned}
\qquad
\Rightarrow aaaaabbQccQQ
$$

Generative Grammars

Example: A grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{aligned}
S &\to aSQ \\
S &\to abc \\
cQ &\to Qc \\
bQc &\to bbcc
\end{aligned}
$$

A derivation of word $aaaaabbbbbccccc$:

$$
\begin{aligned}
S &\Rightarrow aSQ \\
&\Rightarrow aaSQQ \\
&\Rightarrow aaaSQQQ \\
&\Rightarrow aaaaSQQQQ \\
&\Rightarrow aaaaabcQQQQ \\
&\Rightarrow aaaaabQcQQQ \\
&\Rightarrow aaaaabbccQQQ \\
&\Rightarrow aaaaabbcQcQQ \\
&\Rightarrow aaaaabbQccQQ \\
&\Rightarrow aaaaabbbcccQQ
\end{aligned}
$$

Generative Grammars

Example: A grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{aligned}
S &\to aSQ \\
S &\to abc \\
cQ &\to Qc \\
bQc &\to bbcc
\end{aligned}
$$

A derivation of word $aaaaabbbbbccccc$:

$$
\begin{aligned}
S &\Rightarrow aSQ \\
&\Rightarrow aaSQQ \\
&\Rightarrow aaaSQQQ \\
&\Rightarrow aaaaSQQQQ \\
&\Rightarrow aaaaabcQQQQ \\
&\Rightarrow aaaaabQcQQQ \\
&\Rightarrow aaaaabbccQQQ \\
&\Rightarrow aaaaabbcQcQQ \\
&\Rightarrow aaaaabbQccQQ \\
&\Rightarrow aaaaabbbcccQQ \\
&\Rightarrow aaaaabbbccQcQ
\end{aligned}
$$

Generative Grammars

Example: A grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{aligned}
S &\rightarrow aSQ \\
S &\rightarrow abc \\
cQ &\rightarrow Qc \\
bQc &\rightarrow bbcc
\end{aligned}
$$

A derivation of word $aaaaabbbbbccccc$:

$$
\begin{aligned}
S &\Rightarrow aSQ \\
&\Rightarrow aaSQQ \\
&\Rightarrow aaaSQQQ \\
&\Rightarrow aaaaSQQQQ \\
&\Rightarrow aaaaabcQQQQ \\
&\Rightarrow aaaaabQcQQQ \\
&\Rightarrow aaaaabbccQQQ \\
&\Rightarrow aaaaabbcQcQQ
\end{aligned}
$$

$$
\begin{aligned}
&\Rightarrow aaaaabbQccQQ \\
&\Rightarrow aaaaabbbcccQQ \\
&\Rightarrow aaaaabbbccQcQ \\
&\Rightarrow aaaaabbbcQccQ
\end{aligned}
$$

Generative Grammars

Example: A grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abc \\ cQ &\rightarrow Qc \\ bQc &\rightarrow bbcc \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\begin{aligned} S &\Rightarrow aSQ \\ &\Rightarrow aaSQQ \\ &\Rightarrow aaaSQQQ \\ &\Rightarrow aaaaSQQQQ \\ &\Rightarrow aaaaabcQQQQ \\ &\Rightarrow aaaaabQcQQQ \\ &\Rightarrow aaaaabbccQQQ \\ &\Rightarrow aaaaabbcQcQQ \\ &\Rightarrow aaaaabbQccQQ \\ &\Rightarrow aaaaabbbcccQQ \\ &\Rightarrow aaaaabbbccQcQ \\ &\Rightarrow aaaaabbbcQccQ \\ &\Rightarrow aaaaabbbQcccQ \end{aligned}$$

Generative Grammars

Example: A grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abc \\ cQ &\rightarrow Qc \\ bQc &\rightarrow bbcc \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\begin{aligned} S &\Rightarrow aSQ \\ &\Rightarrow aaSQQ \\ &\Rightarrow aaaSQQQ \\ &\Rightarrow aaaaSQQQQ \\ &\Rightarrow aaaaabcQQQQ \\ &\Rightarrow aaaaabQcQQQ \\ &\Rightarrow aaaaabbccQQQ \\ &\Rightarrow aaaaabbcQcQQ \\ &\Rightarrow aaaaabbQccQQ \\ &\Rightarrow aaaaabbbcccQQ \\ &\Rightarrow aaaaabbbccQcQ \\ &\Rightarrow aaaaabbbcQccQ \\ &\Rightarrow aaaaabbbQcccQ \\ &\Rightarrow aaaaabbbbccccQ \end{aligned}$$

Generative Grammars

Example: A grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{aligned}
S &\rightarrow aSQ \\
S &\rightarrow abc \\
cQ &\rightarrow Qc \\
bQc &\rightarrow bbcc
\end{aligned}
$$

A derivation of word $aaaaabbbbbccccc$:

$$
\begin{aligned}
S &\Rightarrow aSQ \\
&\Rightarrow aaSQQ \\
&\Rightarrow aaaSQQQ \\
&\Rightarrow aaaaSQQQQ \\
&\Rightarrow aaaaabcQQQQ \\
&\Rightarrow aaaaabQcQQQ \\
&\Rightarrow aaaaabbccQQQ \\
&\Rightarrow aaaaabbcQcQQ
\end{aligned}
\qquad
\begin{aligned}
&\Rightarrow aaaaabbQccQQ \\
&\Rightarrow aaaaabbbcccQQ \\
&\Rightarrow aaaaabbbccQcQ \\
&\Rightarrow aaaaabbbcQccQ \\
&\Rightarrow aaaaabbbQcccQ \\
&\Rightarrow aaaaabbbbccccQ \\
&\Rightarrow aaaaabbbbcccQc
\end{aligned}
$$

Generative Grammars

Example: A grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{aligned}
S &\to aSQ \\
S &\to abc \\
cQ &\to Qc \\
bQc &\to bbcc
\end{aligned}
$$

A derivation of word $aaaaabbbbbccccc$:

$$
\begin{aligned}
S &\Rightarrow aSQ & &\Rightarrow aaaaabbQccQQ \\
&\Rightarrow aaSQQ & &\Rightarrow aaaaabbbcccQQ \\
&\Rightarrow aaaSQQQ & &\Rightarrow aaaaabbbccQcQ \\
&\Rightarrow aaaaSQQQQ & &\Rightarrow aaaaabbbcQccQ \\
&\Rightarrow aaaaabcQQQQ & &\Rightarrow aaaaabbbQcccQ \\
&\Rightarrow aaaaabQcQQQ & &\Rightarrow aaaaabbbbccccQ \\
&\Rightarrow aaaaabbccQQQ & &\Rightarrow aaaaabbbbcccQc \\
&\Rightarrow aaaaabbcQcQQ & &\Rightarrow aaaaabbbbccQcc
\end{aligned}
$$

Generative Grammars

Example: A grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{aligned}
S &\to aSQ \\
S &\to abc \\
cQ &\to Qc \\
bQc &\to bbcc
\end{aligned}
$$

A derivation of word $aaaabbbbcccc$:

$$\Longrightarrow \ aaaabbbcQccc$$

$$
\begin{aligned}
&\Longrightarrow \ aaaabbQccQQ \\
&\Longrightarrow \ aaaabbbcccQQ \\
&\Longrightarrow \ aaaabbbccQcQ \\
&\Longrightarrow \ aaaabbbcQccQ \\
&\Longrightarrow \ aaaabbbQcccQ \\
&\Longrightarrow \ aaaabbbbccccQ \\
&\Longrightarrow \ aaaabbbbcccQc \\
&\Longrightarrow \ aaaabbbbccQcc
\end{aligned}
$$

Example: A grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abc \\ cQ &\rightarrow Qc \\ bQc &\rightarrow bbcc \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\Rightarrow aaaaabbQccQQ$$
$$\Rightarrow aaaaabbbcccQQ$$
$$\Rightarrow aaaaabbbccQcQ$$
$$\Rightarrow aaaaabbbcQccQ$$
$$\Rightarrow aaaaabbbQcccQ$$
$$\Rightarrow aaaaabbbbccccQ$$
$$\Rightarrow aaaaabbbbcccQc$$
$$\Rightarrow aaaaabbbbccQcc$$
$$\Rightarrow aaaaabbbbcQccc$$
$$\Rightarrow aaaaabbbbQcccc$$

Generative Grammars

Example: A grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abc \\ cQ &\rightarrow Qc \\ bQc &\rightarrow bbcc \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$\Rightarrow\ aaaaabbbbcQccc$

$\Rightarrow\ aaaaabbbbQcccc$

$\Rightarrow\ aaaaabbbbbccccc$

$\Rightarrow\ aaaaabbQccQQ$

$\Rightarrow\ aaaaabbbcccQQ$

$\Rightarrow\ aaaaabbbccQcQ$

$\Rightarrow\ aaaaabbbcQccQ$

$\Rightarrow\ aaaaabbbQcccQ$

$\Rightarrow\ aaaaabbbbccccQ$

$\Rightarrow\ aaaaabbbbcccQc$

$\Rightarrow\ aaaaabbbbccQcc$

Context-sensitive Grammars

Context-sensitive grammars, also called **type-1** grammars, are a special case of generative grammars.

A grammar $\mathcal{G} = (\Pi, \Sigma, S, P)$ is called **context-sensitive** if all its rules (with one exception given below) are of the form

$$\alpha X \beta \rightarrow \alpha \gamma \beta$$

where $X \in \Pi$, $\alpha, \beta, \gamma \in (\Pi \cup \Sigma)^*$, with $|\gamma| \geq 1$.

The only exception is that the grammar can contain the rule $S \rightarrow \varepsilon$.

If $\mathcal{G}$ contains this rule then the initial nonterminal S can not occur on the right-hand side of any rule.

An example of a rule:

$$BaEC \rightarrow BaDAcBC$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\to aSQ \\ S &\to abC \\ bQC &\to bbCC \\ C &\to c \end{aligned} \qquad \begin{aligned} CQ &\to XQ \\ XQ &\to XY \\ XY &\to QY \\ QY &\to QC \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$S$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abC \\ bQC &\rightarrow bbCC \\ C &\rightarrow c \end{aligned} \qquad\qquad \begin{aligned} CQ &\rightarrow XQ \\ XQ &\rightarrow XY \\ XY &\rightarrow QY \\ QY &\rightarrow QC \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$S \Rightarrow aSQ$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{array}{rcl}
S & \to & aSQ \\
S & \to & abC \\
bQC & \to & bbCC \\
C & \to & c
\end{array}
\qquad\qquad
\begin{array}{rcl}
CQ & \to & XQ \\
XQ & \to & XY \\
XY & \to & QY \\
QY & \to & QC
\end{array}
$$

A derivation of word $aaaaabbbbbccccc$:

$$
\begin{array}{rl}
S \Rightarrow & aSQ \\
\Rightarrow & aaSQQ
\end{array}
$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abC \\ bQC &\rightarrow bbCC \\ C &\rightarrow c \end{aligned} \qquad \begin{aligned} CQ &\rightarrow XQ \\ XQ &\rightarrow XY \\ XY &\rightarrow QY \\ QY &\rightarrow QC \end{aligned}$$

A derivation of word $aaaabbbbcccc$:

$$\begin{aligned} S &\Rightarrow aSQ \\ &\Rightarrow aaSQQ \\ &\Rightarrow aaaSQQQ \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abC \\ bQC &\rightarrow bbCC \\ C &\rightarrow c \end{aligned} \qquad \begin{aligned} CQ &\rightarrow XQ \\ XQ &\rightarrow XY \\ XY &\rightarrow QY \\ QY &\rightarrow QC \end{aligned}$$

A derivation of word $aaaabbbbcccc$:

$$\begin{aligned} S &\Rightarrow aSQ \\ &\Rightarrow aaSQQ \\ &\Rightarrow aaaSQQQ \\ &\Rightarrow aaaaSQQQQ \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{aligned}
S &\rightarrow aSQ \\
S &\rightarrow abC \\
bQC &\rightarrow bbCC \\
C &\rightarrow c
\end{aligned}
\qquad\qquad
\begin{aligned}
CQ &\rightarrow XQ \\
XQ &\rightarrow XY \\
XY &\rightarrow QY \\
QY &\rightarrow QC
\end{aligned}
$$

A derivation of word $aaaaabbbbbccccc$:

$$
\begin{aligned}
S &\Rightarrow aSQ \\
&\Rightarrow aaSQQ \\
&\Rightarrow aaaSQQQ \\
&\Rightarrow aaaaSQQQQ \\
&\Rightarrow aaaaabCQQQQ
\end{aligned}
$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ & CQ &\rightarrow XQ \\ S &\rightarrow abC & XQ &\rightarrow XY \\ bQC &\rightarrow bbCC & XY &\rightarrow QY \\ C &\rightarrow c & QY &\rightarrow QC \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\begin{aligned} S &\Rightarrow aSQ \\ &\Rightarrow aaSQQ \\ &\Rightarrow aaaSQQQ \\ &\Rightarrow aaaaSQQQQ \\ &\Rightarrow aaaaabCQQQQ \\ &\Rightarrow aaaaabXQQQQ \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abC \\ bQC &\rightarrow bbCC \\ C &\rightarrow c \end{aligned} \qquad\qquad \begin{aligned} CQ &\rightarrow XQ \\ XQ &\rightarrow XY \\ XY &\rightarrow QY \\ QY &\rightarrow QC \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\begin{aligned} S &\Rightarrow aSQ \\ &\Rightarrow aaSQQ \\ &\Rightarrow aaaSQQQ \\ &\Rightarrow aaaaSQQQQ \\ &\Rightarrow aaaaabCQQQQ \\ &\Rightarrow aaaaabXQQQQ \\ &\Rightarrow aaaaabXYQQQ \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\to aSQ & CQ &\to XQ \\ S &\to abC & XQ &\to XY \\ bQC &\to bbCC & XY &\to QY \\ C &\to c & QY &\to QC \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\begin{aligned} S &\Rightarrow aSQ \\ &\Rightarrow aaSQQ \\ &\Rightarrow aaaSQQQ \\ &\Rightarrow aaaaSQQQQ \\ &\Rightarrow aaaaabCQQQQ \\ &\Rightarrow aaaaabXQQQQ \\ &\Rightarrow aaaaabXYQQQ \\ &\Rightarrow aaaaabQYQQQ \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{aligned}
S &\rightarrow aSQ & CQ &\rightarrow XQ \\
S &\rightarrow abC & XQ &\rightarrow XY \\
bQC &\rightarrow bbCC & XY &\rightarrow QY \\
C &\rightarrow c & QY &\rightarrow QC
\end{aligned}
$$

A derivation of word $aaaaabbbbbccccc$:

$$
\begin{aligned}
S &\Rightarrow aSQ \\
&\Rightarrow aaSQQ \\
&\Rightarrow aaaSQQQ \\
&\Rightarrow aaaaSQQQQ \\
&\Rightarrow aaaaabCQQQQ \\
&\Rightarrow aaaaabXQQQQ \\
&\Rightarrow aaaaabXYQQQ \\
&\Rightarrow aaaaabQYQQQ \\
&\Rightarrow aaaaabQCQQQ
\end{aligned}
$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\to aSQ \\ S &\to abC \\ bQC &\to bbCC \\ C &\to c \end{aligned} \qquad\qquad \begin{aligned} CQ &\to XQ \\ XQ &\to XY \\ XY &\to QY \\ QY &\to QC \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\begin{aligned} S &\Rightarrow aSQ \\ &\Rightarrow aaSQQ \\ &\Rightarrow aaaSQQQ \\ &\Rightarrow aaaaSQQQQ \\ &\Rightarrow aaaaabCQQQQ \\ &\Rightarrow aaaaabXQQQQ \\ &\Rightarrow aaaaabXYQQQ \\ &\Rightarrow aaaaabQYQQQ \end{aligned} \qquad\qquad \begin{aligned} &\Rightarrow aaaaabQCQQQ \\ &\Rightarrow aaaaabbCCQQQ \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abC \\ bQC &\rightarrow bbCC \\ C &\rightarrow c \end{aligned} \qquad\qquad \begin{aligned} CQ &\rightarrow XQ \\ XQ &\rightarrow XY \\ XY &\rightarrow QY \\ QY &\rightarrow QC \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\begin{aligned} S &\Rightarrow aSQ \\ &\Rightarrow aaSQQ \\ &\Rightarrow aaaSQQQ \\ &\Rightarrow aaaaSQQQQ \\ &\Rightarrow aaaaabCQQQQ \\ &\Rightarrow aaaaabXQQQQ \\ &\Rightarrow aaaaabXYQQQ \\ &\Rightarrow aaaaabQYQQQ \end{aligned}$$

$$\begin{aligned} &\Rightarrow aaaaabQCQQQ \\ &\Rightarrow aaaaabbCCQQQ \\ &\Rightarrow aaaaabbCXQQQ \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{aligned}
S &\to aSQ & \qquad CQ &\to XQ \\
S &\to abC & \qquad XQ &\to XY \\
bQC &\to bbCC & \qquad XY &\to QY \\
C &\to c & \qquad QY &\to QC
\end{aligned}
$$

A derivation of word $aaaaabbbbbccccc$:

$$
\begin{aligned}
S &\Rightarrow aSQ \\
&\Rightarrow aaSQQ \\
&\Rightarrow aaaSQQQ \\
&\Rightarrow aaaaSQQQQ \\
&\Rightarrow aaaaabCQQQQ \\
&\Rightarrow aaaaabXQQQQ \\
&\Rightarrow aaaaabXYQQQ \\
&\Rightarrow aaaaabQYQQQ \\
&\Rightarrow aaaaabQCQQQ \\
&\Rightarrow aaaaabbCCQQQ \\
&\Rightarrow aaaaabbCXQQQ \\
&\Rightarrow aaaaabbCXYQQ
\end{aligned}
$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\to aSQ \\ S &\to abC \\ bQC &\to bbCC \\ C &\to c \end{aligned} \qquad \begin{aligned} CQ &\to XQ \\ XQ &\to XY \\ XY &\to QY \\ QY &\to QC \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\begin{aligned} S &\Rightarrow aSQ \\ &\Rightarrow aaSQQ \\ &\Rightarrow aaaSQQQ \\ &\Rightarrow aaaaSQQQQ \\ &\Rightarrow aaaaabCQQQQ \\ &\Rightarrow aaaaabXQQQQ \\ &\Rightarrow aaaaabXYQQQ \\ &\Rightarrow aaaaabQYQQQ \end{aligned} \qquad \begin{aligned} &\Rightarrow aaaaabQCQQQ \\ &\Rightarrow aaaaabbCCQQQ \\ &\Rightarrow aaaaabbCXQQQ \\ &\Rightarrow aaaaabbCXYQQ \\ &\Rightarrow aaaaabbCQYQQ \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abC \\ bQC &\rightarrow bbCC \\ C &\rightarrow c \end{aligned} \qquad\qquad \begin{aligned} CQ &\rightarrow XQ \\ XQ &\rightarrow XY \\ XY &\rightarrow QY \\ QY &\rightarrow QC \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\begin{aligned} S &\Rightarrow aSQ \\ &\Rightarrow aaSQQ \\ &\Rightarrow aaaSQQQ \\ &\Rightarrow aaaaSQQQQ \\ &\Rightarrow aaaaabCQQQQ \\ &\Rightarrow aaaaabXQQQQ \\ &\Rightarrow aaaaabXYQQQ \\ &\Rightarrow aaaaabQYQQQ \end{aligned} \qquad \begin{aligned} &\Rightarrow aaaaabQCQQQ \\ &\Rightarrow aaaaabbCCQQQ \\ &\Rightarrow aaaaabbCXQQQ \\ &\Rightarrow aaaaabbCXYQQ \\ &\Rightarrow aaaaabbCQYQQ \\ &\Rightarrow aaaaabbCQCQQ \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{aligned}
S &\rightarrow aSQ & \qquad CQ &\rightarrow XQ \\
S &\rightarrow abC & XQ &\rightarrow XY \\
bQC &\rightarrow bbCC & XY &\rightarrow QY \\
C &\rightarrow c & QY &\rightarrow QC
\end{aligned}
$$

A derivation of word $aaaaabbbbbccccc$:

$$
\begin{aligned}
S &\Rightarrow aSQ & &\Rightarrow aaaaabQCQQQ \\
&\Rightarrow aaSQQ & &\Rightarrow aaaaabbCCQQQ \\
&\Rightarrow aaaSQQQ & &\Rightarrow aaaaabbCXQQQ \\
&\Rightarrow aaaaSQQQQ & &\Rightarrow aaaaabbCXYQQ \\
&\Rightarrow aaaaabCQQQQ & &\Rightarrow aaaaabbCQYQQ \\
&\Rightarrow aaaaabXQQQQ & &\Rightarrow aaaaabbCQCQQ \\
&\Rightarrow aaaaabXYQQQ & &\Rightarrow aaaaabbXQCQQ \\
&\Rightarrow aaaaabQYQQQ & &
\end{aligned}
$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\to aSQ & \qquad CQ &\to XQ \\ S &\to abC & XQ &\to XY \\ bQC &\to bbCC & XY &\to QY \\ C &\to c & QY &\to QC \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\begin{aligned} S &\Rightarrow aSQ \\ &\Rightarrow aaSQQ \\ &\Rightarrow aaaSQQQ \\ &\Rightarrow aaaaSQQQQ \\ &\Rightarrow aaaaabCQQQQ \\ &\Rightarrow aaaaabXQQQQ \\ &\Rightarrow aaaaabXYQQQ \\ &\Rightarrow aaaaabQYQQQ \end{aligned}$$

$$\begin{aligned} &\Rightarrow aaaaabQCQQQ \\ &\Rightarrow aaaaabbCCQQQ \\ &\Rightarrow aaaaabbCXQQQ \\ &\Rightarrow aaaaabbCXYQQ \\ &\Rightarrow aaaaabbCQYQQ \\ &\Rightarrow aaaaabbCQCQQ \\ &\Rightarrow aaaaabbXQCQQ \\ &\Rightarrow aaaaabbXYCQQ \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\to aSQ \\ S &\to abC \\ bQC &\to bbCC \\ C &\to c \end{aligned} \qquad\qquad \begin{aligned} CQ &\to XQ \\ XQ &\to XY \\ XY &\to QY \\ QY &\to QC \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\Rightarrow\ aaaaabbQYCQQ$$

$$\begin{aligned} &\Rightarrow\ aaaaabQCQQQ \\ &\Rightarrow\ aaaaabbCCQQQ \\ &\Rightarrow\ aaaaabbCXQQQ \\ &\Rightarrow\ aaaaabbCXYQQ \\ &\Rightarrow\ aaaaabbCQYQQ \\ &\Rightarrow\ aaaaabbCQCQQ \\ &\Rightarrow\ aaaaabbXQCQQ \\ &\Rightarrow\ aaaaabbXYCQQ \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\to aSQ \\ S &\to abC \\ bQC &\to bbCC \\ C &\to c \end{aligned} \qquad\qquad \begin{aligned} CQ &\to XQ \\ XQ &\to XY \\ XY &\to QY \\ QY &\to QC \end{aligned}$$

A derivation of word $aaaabbbbcccc$:

$$\Rightarrow aaaaabbQYCQQ$$
$$\Rightarrow aaaaabbQCCQQ$$

$$\Rightarrow aaaaabQCQQQ$$
$$\Rightarrow aaaaabbCCQQQ$$
$$\Rightarrow aaaaabbCXQQQ$$
$$\Rightarrow aaaaabbCXYQQ$$
$$\Rightarrow aaaaabbCQYQQ$$
$$\Rightarrow aaaaabbCQCQQ$$
$$\Rightarrow aaaaabbXQCQQ$$
$$\Rightarrow aaaaabbXYCQQ$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{aligned}
S &\to aSQ & \qquad CQ &\to XQ \\
S &\to abC & XQ &\to XY \\
bQC &\to bbCC & XY &\to QY \\
C &\to c & QY &\to QC
\end{aligned}
$$

A derivation of word $aaaaabbbbbccccc$:

$$
\begin{aligned}
&\Rightarrow aaaaabQCQQQ \\
&\Rightarrow aaaaabbCCQQQ \\
&\Rightarrow aaaaabbCXQQQ \\
&\Rightarrow aaaaabbCXYQQ \\
&\Rightarrow aaaaabbCQYQQ \\
&\Rightarrow aaaaabbCQCQQ \\
&\Rightarrow aaaaabbXQCQQ \\
&\Rightarrow aaaaabbXYCQQ \\
&\Rightarrow aaaaabbQYCQQ \\
&\Rightarrow aaaaabbQCCQQ \\
&\Rightarrow aaaaabbbCCCQQ
\end{aligned}
$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\to aSQ \\ S &\to abC \\ bQC &\to bbCC \\ C &\to c \end{aligned} \qquad \begin{aligned} CQ &\to XQ \\ XQ &\to XY \\ XY &\to QY \\ QY &\to QC \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\begin{aligned} &\Rightarrow aaaaabbQYCQQ \\ &\Rightarrow aaaaabbQCCQQ \\ &\Rightarrow aaaaabbbCCCQQ \\ &\Rightarrow aaaaabbbCCXQQ \end{aligned}$$

$$\begin{aligned} &\Rightarrow aaaaabQCQQQ \\ &\Rightarrow aaaaabbCCQQQ \\ &\Rightarrow aaaaabbCXQQQ \\ &\Rightarrow aaaaabbCXYQQ \\ &\Rightarrow aaaaabbCQYQQ \\ &\Rightarrow aaaaabbCQCQQ \\ &\Rightarrow aaaaabbXQCQQ \\ &\Rightarrow aaaaabbXYCQQ \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{aligned}
S &\rightarrow aSQ & CQ &\rightarrow XQ \\
S &\rightarrow abC & XQ &\rightarrow XY \\
bQC &\rightarrow bbCC & XY &\rightarrow QY \\
C &\rightarrow c & QY &\rightarrow QC
\end{aligned}
$$

A derivation of word $aaaaabbbbbccccc$:

$$
\begin{aligned}
&\Rightarrow aaaaabbQYCQQ \\
&\Rightarrow aaaaabbQCCQQ \\
&\Rightarrow aaaaabbbCCCQQ \\
&\Rightarrow aaaaabbbCCXQQ \\
&\Rightarrow aaaaabbbCCXYQ
\end{aligned}
$$

$$
\begin{aligned}
&\Rightarrow aaaaabQCQQQ \\
&\Rightarrow aaaaabbCCQQQ \\
&\Rightarrow aaaaabbCXQQQ \\
&\Rightarrow aaaaabbCXYQQ \\
&\Rightarrow aaaaabbCQYQQ \\
&\Rightarrow aaaaabbCQCQQ \\
&\Rightarrow aaaaabbXQCQQ \\
&\Rightarrow aaaaabbXYCQQ
\end{aligned}
$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{array}{rcl}
S & \rightarrow & aSQ \\
S & \rightarrow & abC \\
bQC & \rightarrow & bbCC \\
C & \rightarrow & c
\end{array}
\qquad
\begin{array}{rcl}
CQ & \rightarrow & XQ \\
XQ & \rightarrow & XY \\
XY & \rightarrow & QY \\
QY & \rightarrow & QC
\end{array}
$$

A derivation of word $aaaabbbbcccc$:

$$
\begin{array}{l}
\Rightarrow\ aaaaabbQYCQQ \\
\Rightarrow\ aaaaabbQCCQQ \\
\Rightarrow\ aaaaabbbCCCQQ \\
\Rightarrow\ aaaaabbbCCXQQ \\
\Rightarrow\ aaaaabbbCCXYQ \\
\Rightarrow\ aaaaabbbCCQYQ
\end{array}
\qquad
\begin{array}{l}
\Rightarrow\ aaaaabQCQQQ \\
\Rightarrow\ aaaaabbCCQQQ \\
\Rightarrow\ aaaaabbCXQQQ \\
\Rightarrow\ aaaaabbCXYQQ \\
\Rightarrow\ aaaaabbCQYQQ \\
\Rightarrow\ aaaaabbCQCQQ \\
\Rightarrow\ aaaaabbXQCQQ \\
\Rightarrow\ aaaaabbXYCQQ
\end{array}
$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{aligned}
S &\to aSQ & \qquad CQ &\to XQ \\
S &\to abC & \qquad XQ &\to XY \\
bQC &\to bbCC & \qquad XY &\to QY \\
C &\to c & \qquad QY &\to QC
\end{aligned}
$$

A derivation of word $aaaabbbbcccc$:

$$
\begin{aligned}
&\Rightarrow aaaaabbQYCQQ \\
&\Rightarrow aaaaabbQCCQQ \\
&\Rightarrow aaaaabbbCCCQQ \\
&\Rightarrow aaaaabbbCCXQQ \\
&\Rightarrow aaaaabbbCCXYQ \\
&\Rightarrow aaaaabbbCCQYQ \\
&\Rightarrow aaaaabbbCCQCQ
\end{aligned}
$$

$$
\begin{aligned}
&\Rightarrow aaaaabQCQQQ \\
&\Rightarrow aaaaabbCCQQQ \\
&\Rightarrow aaaaabbCXQQQ \\
&\Rightarrow aaaaabbCXYQQ \\
&\Rightarrow aaaaabbCQYQQ \\
&\Rightarrow aaaaabbCQCQQ \\
&\Rightarrow aaaaabbXQCQQ \\
&\Rightarrow aaaaabbXYCQQ
\end{aligned}
$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{aligned}
S &\to aSQ & \qquad CQ &\to XQ \\
S &\to abC & XQ &\to XY \\
bQC &\to bbCC & XY &\to QY \\
C &\to c & QY &\to QC
\end{aligned}
$$

A derivation of word $aaaaabbbbbccccc$:

$$
\begin{aligned}
&\Rightarrow aaaaabbQYCQQ \\
&\Rightarrow aaaaabbQCCQQ \\
&\Rightarrow aaaaabbbCCCQQ \\
&\Rightarrow aaaaabbbCCXQQ \\
&\Rightarrow aaaaabbbCCXYQ \\
&\Rightarrow aaaaabbbCCQYQ \\
&\Rightarrow aaaaabbbCCQCQ \\
&\Rightarrow aaaaabbbCXQCQ
\end{aligned}
\qquad
\begin{aligned}
&\Rightarrow aaaaabQCQQQ \\
&\Rightarrow aaaaabbCCQQQ \\
&\Rightarrow aaaaabbCXQQQ \\
&\Rightarrow aaaaabbCXYQQ \\
&\Rightarrow aaaaabbCQYQQ \\
&\Rightarrow aaaaabbCQCQQ \\
&\Rightarrow aaaaabbXQCQQ \\
&\Rightarrow aaaaabbXYCQQ
\end{aligned}
$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\to aSQ \\ S &\to abC \\ bQC &\to bbCC \\ C &\to c \end{aligned} \qquad\qquad \begin{aligned} CQ &\to XQ \\ XQ &\to XY \\ XY &\to QY \\ QY &\to QC \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\begin{aligned} &\Rightarrow aaaaabbQYCQQ \\ &\Rightarrow aaaaabbQCCQQ \\ &\Rightarrow aaaaabbbCCCQQ \\ &\Rightarrow aaaaabbbCCXQQ \\ &\Rightarrow aaaaabbbCCXYQ \\ &\Rightarrow aaaaabbbCCQYQ \\ &\Rightarrow aaaaabbbCCQCQ \\ &\Rightarrow aaaaabbbCXQCQ \end{aligned} \qquad\qquad \begin{aligned} &\Rightarrow aaaaabbbCXYCQ \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$S \rightarrow aSQ \qquad CQ \rightarrow XQ$$
$$S \rightarrow abC \qquad XQ \rightarrow XY$$
$$bQC \rightarrow bbCC \qquad XY \rightarrow QY$$
$$C \rightarrow c \qquad QY \rightarrow QC$$

A derivation of word $aaaabbbbcccc$:

$\Rightarrow\ aaaaabbQYCQQ$
$\Rightarrow\ aaaaabbQCCQQ$
$\Rightarrow\ aaaaabbbCCCQQ$
$\Rightarrow\ aaaaabbbCCXQQ$
$\Rightarrow\ aaaaabbbCCXYQ$
$\Rightarrow\ aaaaabbbCCQYQ$
$\Rightarrow\ aaaaabbbCCQCQ$
$\Rightarrow\ aaaaabbbCXQCQ$
$\Rightarrow\ aaaaabbbCXYCQ$
$\Rightarrow\ aaaaabbbCQYCQ$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abC \\ bQC &\rightarrow bbCC \\ C &\rightarrow c \end{aligned} \qquad\qquad \begin{aligned} CQ &\rightarrow XQ \\ XQ &\rightarrow XY \\ XY &\rightarrow QY \\ QY &\rightarrow QC \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$\Rightarrow aaaaabbQYCQQ$
$\Rightarrow aaaaabbQCCQQ$
$\Rightarrow aaaaabbbCCCQQ$
$\Rightarrow aaaaabbbCCXQQ$
$\Rightarrow aaaaabbbCCXYQ$
$\Rightarrow aaaaabbbCCQYQ$
$\Rightarrow aaaaabbbCCQCQ$
$\Rightarrow aaaaabbbCXQCQ$
$\Rightarrow aaaaabbbCXYCQ$
$\Rightarrow aaaaabbbCQYCQ$
$\Rightarrow aaaaabbbCQCCQ$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\to aSQ & CQ &\to XQ \\ S &\to abC & XQ &\to XY \\ bQC &\to bbCC & XY &\to QY \\ C &\to c & QY &\to QC \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\begin{aligned} &\Rightarrow aaaaabbQYCQQ \\ &\Rightarrow aaaaabbQCCQQ \\ &\Rightarrow aaaaabbbCCCQQ \\ &\Rightarrow aaaaabbbCCXQQ \\ &\Rightarrow aaaaabbbCCXYQ \\ &\Rightarrow aaaaabbbCCQYQ \\ &\Rightarrow aaaaabbbCCQCQ \\ &\Rightarrow aaaaabbbCXQCQ \end{aligned}$$

$$\begin{aligned} &\Rightarrow aaaaabbbCXYCQ \\ &\Rightarrow aaaaabbbCQYCQ \\ &\Rightarrow aaaaabbbCQCCQ \\ &\Rightarrow aaaaabbbXQCCQ \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{aligned}
S &\to aSQ \\
S &\to abC \\
bQC &\to bbCC \\
C &\to c
\end{aligned}
\qquad\qquad
\begin{aligned}
CQ &\to XQ \\
XQ &\to XY \\
XY &\to QY \\
QY &\to QC
\end{aligned}
$$

A derivation of word $aaaaabbbbbccccc$:

$$
\begin{aligned}
&\Rightarrow aaaaabbQYCQQ \\
&\Rightarrow aaaaabbQCCQQ \\
&\Rightarrow aaaaabbbCCCQQ \\
&\Rightarrow aaaaabbbCCXQQ \\
&\Rightarrow aaaaabbbCCXYQ \\
&\Rightarrow aaaaabbbCCQYQ \\
&\Rightarrow aaaaabbbCCQCQ \\
&\Rightarrow aaaaabbbCXQCQ
\end{aligned}
\qquad\qquad
\begin{aligned}
&\Rightarrow aaaaabbbCXYCQ \\
&\Rightarrow aaaaabbbCQYCQ \\
&\Rightarrow aaaaabbbCQCCQ \\
&\Rightarrow aaaaabbbXQCCQ \\
&\Rightarrow aaaaabbbXYCCQ
\end{aligned}
$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abC \\ bQC &\rightarrow bbCC \\ C &\rightarrow c \end{aligned} \qquad \begin{aligned} CQ &\rightarrow XQ \\ XQ &\rightarrow XY \\ XY &\rightarrow QY \\ QY &\rightarrow QC \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\begin{aligned} &\Longrightarrow aaaaabbQYCQQ \\ &\Longrightarrow aaaaabbQCCQQ \\ &\Longrightarrow aaaaabbbCCCQQ \\ &\Longrightarrow aaaaabbbCCXQQ \\ &\Longrightarrow aaaaabbbCCXYQ \\ &\Longrightarrow aaaaabbbCCQYQ \\ &\Longrightarrow aaaaabbbCCQCQ \\ &\Longrightarrow aaaaabbbCXQCQ \end{aligned} \qquad \begin{aligned} &\Longrightarrow aaaaabbbCXYCQ \\ &\Longrightarrow aaaaabbbCQYCQ \\ &\Longrightarrow aaaaabbbCQCCQ \\ &\Longrightarrow aaaaabbbXQCCQ \\ &\Longrightarrow aaaaabbbXYCCQ \\ &\Longrightarrow aaaaabbbQYCCQ \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abC \\ bQC &\rightarrow bbCC \\ C &\rightarrow c \end{aligned} \qquad \begin{aligned} CQ &\rightarrow XQ \\ XQ &\rightarrow XY \\ XY &\rightarrow QY \\ QY &\rightarrow QC \end{aligned}$$

A derivation of word $aaaabbbbcccc$:

$$\begin{aligned} &\Rightarrow aaaaabbQYCQQ \\ &\Rightarrow aaaaabbQCCQQ \\ &\Rightarrow aaaaabbbCCCQQ \\ &\Rightarrow aaaaabbbCCXQQ \\ &\Rightarrow aaaaabbbCCXYQ \\ &\Rightarrow aaaaabbbCCQYQ \\ &\Rightarrow aaaaabbbCCQCQ \\ &\Rightarrow aaaaabbbCXQCQ \end{aligned} \qquad \begin{aligned} &\Rightarrow aaaaabbbCXYCQ \\ &\Rightarrow aaaaabbbCQYCQ \\ &\Rightarrow aaaaabbbCQCCQ \\ &\Rightarrow aaaaabbbXQCCQ \\ &\Rightarrow aaaaabbbXYCCQ \\ &\Rightarrow aaaaabbbQYCCQ \\ &\Rightarrow aaaaabbbQCCCQ \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$S \rightarrow aSQ \qquad CQ \rightarrow XQ$$
$$S \rightarrow abC \qquad XQ \rightarrow XY$$
$$bQC \rightarrow bbCC \qquad XY \rightarrow QY$$
$$C \rightarrow c \qquad QY \rightarrow QC$$

A derivation of word $aaaaabbbbbccccc$:

$$\Longrightarrow aaaaabbQYCQQ$$
$$\Longrightarrow aaaaabbQCCQQ$$
$$\Longrightarrow aaaaabbbCCCQQ$$
$$\Longrightarrow aaaaabbbCCXQQ$$
$$\Longrightarrow aaaaabbbCCXYQ$$
$$\Longrightarrow aaaaabbbCCQYQ$$
$$\Longrightarrow aaaaabbbCCQCQ$$
$$\Longrightarrow aaaaabbbCXQCQ$$
$$\Longrightarrow aaaaabbbCXYCQ$$
$$\Longrightarrow aaaaabbbCQYCQ$$
$$\Longrightarrow aaaaabbbCQCCQ$$
$$\Longrightarrow aaaaabbbXQCCQ$$
$$\Longrightarrow aaaaabbbXYCCQ$$
$$\Longrightarrow aaaaabbbQYCCQ$$
$$\Longrightarrow aaaaabbbQCCCQ$$
$$\Longrightarrow aaaaabbbbCCCCQ$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{aligned}
S &\to aSQ \\
S &\to abC \\
bQC &\to bbCC \\
C &\to c
\end{aligned}
\qquad\qquad
\begin{aligned}
CQ &\to XQ \\
XQ &\to XY \\
XY &\to QY \\
QY &\to QC
\end{aligned}
$$

A derivation of word $aaaabbbbcccc$:

$$\Longrightarrow\ aaaaabbbbCCCXQ$$

$$
\begin{aligned}
&\Longrightarrow\ aaaaabbbCXYCQ \\
&\Longrightarrow\ aaaaabbbCQYCQ \\
&\Longrightarrow\ aaaaabbbCQCCQ \\
&\Longrightarrow\ aaaaabbbXQCCQ \\
&\Longrightarrow\ aaaaabbbXYCCQ \\
&\Longrightarrow\ aaaaabbbQYCCQ \\
&\Longrightarrow\ aaaaabbbQCCCQ \\
&\Longrightarrow\ aaaaabbbbCCCCQ
\end{aligned}
$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abC \\ bQC &\rightarrow bbCC \\ C &\rightarrow c \end{aligned} \qquad\qquad \begin{aligned} CQ &\rightarrow XQ \\ XQ &\rightarrow XY \\ XY &\rightarrow QY \\ QY &\rightarrow QC \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\begin{aligned} &\Rightarrow aaaaabbbbCCCXQ \\ &\Rightarrow aaaaabbbbCCCXY \end{aligned}$$

$$\begin{aligned} &\Rightarrow aaaaabbbCXYCQ \\ &\Rightarrow aaaaabbbCQYCQ \\ &\Rightarrow aaaaabbbCQCCQ \\ &\Rightarrow aaaaabbbXQCCQ \\ &\Rightarrow aaaaabbbXYCCQ \\ &\Rightarrow aaaaabbbQYCCQ \\ &\Rightarrow aaaaabbbQCCCQ \\ &\Rightarrow aaaaabbbbCCCCQ \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abC \\ bQC &\rightarrow bbCC \\ C &\rightarrow c \end{aligned} \qquad \begin{aligned} CQ &\rightarrow XQ \\ XQ &\rightarrow XY \\ XY &\rightarrow QY \\ QY &\rightarrow QC \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\begin{aligned} &\Rightarrow aaaaabbbbCCCXQ \\ &\Rightarrow aaaaabbbbCCCXY \\ &\Rightarrow aaaaabbbbCCCQY \end{aligned}$$

$$\begin{aligned} &\Rightarrow aaaaabbbCXYCQ \\ &\Rightarrow aaaaabbbCQYCQ \\ &\Rightarrow aaaaabbbCQCCQ \\ &\Rightarrow aaaaabbbXQCCQ \\ &\Rightarrow aaaaabbbXYCCQ \\ &\Rightarrow aaaaabbbQYCCQ \\ &\Rightarrow aaaaabbbQCCCQ \\ &\Rightarrow aaaaabbbbCCCCQ \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{aligned}
S &\rightarrow aSQ \\
S &\rightarrow abC \\
bQC &\rightarrow bbCC \\
C &\rightarrow c
\end{aligned}
\qquad\qquad
\begin{aligned}
CQ &\rightarrow XQ \\
XQ &\rightarrow XY \\
XY &\rightarrow QY \\
QY &\rightarrow QC
\end{aligned}
$$

A derivation of word $aaaaabbbbbccccc$:

$$
\begin{aligned}
&\Rightarrow aaaaabbbbCCCXQ \\
&\Rightarrow aaaaabbbbCCCXY \\
&\Rightarrow aaaaabbbbCCCQY \\
&\Rightarrow aaaaabbbbCCCQC
\end{aligned}
$$

$$
\begin{aligned}
&\Rightarrow aaaaabbbCXYCQ \\
&\Rightarrow aaaaabbbCQYCQ \\
&\Rightarrow aaaaabbbCQCCQ \\
&\Rightarrow aaaaabbbXQCCQ \\
&\Rightarrow aaaaabbbXYCCQ \\
&\Rightarrow aaaaabbbQYCCQ \\
&\Rightarrow aaaaabbbQCCCQ \\
&\Rightarrow aaaaabbbbCCCCQ
\end{aligned}
$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\to aSQ & CQ &\to XQ \\ S &\to abC & XQ &\to XY \\ bQC &\to bbCC & XY &\to QY \\ C &\to c & QY &\to QC \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\begin{aligned} &\Rightarrow aaaaabbbbCCCXQ \\ &\Rightarrow aaaaabbbbCCCXY \\ &\Rightarrow aaaaabbbbCCCQY \\ &\Rightarrow aaaaabbbbCCCQC \\ &\Rightarrow aaaaabbbbCCXQC \end{aligned}$$

$$\begin{aligned} &\Rightarrow aaaaabbbCXYCQ \\ &\Rightarrow aaaaabbbCQYCQ \\ &\Rightarrow aaaaabbbCQCCQ \\ &\Rightarrow aaaaabbbXQCCQ \\ &\Rightarrow aaaaabbbXYCCQ \\ &\Rightarrow aaaaabbbQYCCQ \\ &\Rightarrow aaaaabbbQCCCQ \\ &\Rightarrow aaaaabbbbCCCCQ \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abC \\ bQC &\rightarrow bbCC \\ C &\rightarrow c \end{aligned} \qquad\qquad \begin{aligned} CQ &\rightarrow XQ \\ XQ &\rightarrow XY \\ XY &\rightarrow QY \\ QY &\rightarrow QC \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\begin{aligned} &\Longrightarrow aaaaabbbbCCCXQ \\ &\Longrightarrow aaaaabbbbCCCXY \\ &\Longrightarrow aaaaabbbbCCCQY \\ &\Longrightarrow aaaaabbbbCCCQC \\ &\Longrightarrow aaaaabbbbCCXQC \\ &\Longrightarrow aaaaabbbbCCXYC \end{aligned} \qquad\qquad \begin{aligned} &\Longrightarrow aaaaabbbCXYCQ \\ &\Longrightarrow aaaaabbbCQYCQ \\ &\Longrightarrow aaaaabbbCQCCQ \\ &\Longrightarrow aaaaabbbXQCCQ \\ &\Longrightarrow aaaaabbbXYCCQ \\ &\Longrightarrow aaaaabbbQYCCQ \\ &\Longrightarrow aaaaabbbQCCCQ \\ &\Longrightarrow aaaaabbbbCCCCQ \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\to aSQ \\ S &\to abC \\ bQC &\to bbCC \\ C &\to c \end{aligned} \qquad \begin{aligned} CQ &\to XQ \\ XQ &\to XY \\ XY &\to QY \\ QY &\to QC \end{aligned}$$

A derivation of word $aaaabbbbcccc$:

$$\begin{aligned} &\Rightarrow aaaaabbbbCCCXQ \\ &\Rightarrow aaaaabbbbCCCXY \\ &\Rightarrow aaaaabbbbCCCQY \\ &\Rightarrow aaaaabbbbCCCQC \\ &\Rightarrow aaaaabbbbCCXQC \\ &\Rightarrow aaaaabbbbCCXYC \\ &\Rightarrow aaaaabbbbCCQYC \end{aligned} \qquad \begin{aligned} &\Rightarrow aaaaabbbCXYCQ \\ &\Rightarrow aaaaabbbCQYCQ \\ &\Rightarrow aaaaabbbCQCCQ \\ &\Rightarrow aaaaabbbXQCCQ \\ &\Rightarrow aaaaabbbXYCCQ \\ &\Rightarrow aaaaabbbQYCCQ \\ &\Rightarrow aaaaabbbQCCCQ \\ &\Rightarrow aaaaabbbbCCCCQ \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abC \\ bQC &\rightarrow bbCC \\ C &\rightarrow c \end{aligned} \qquad\qquad \begin{aligned} CQ &\rightarrow XQ \\ XQ &\rightarrow XY \\ XY &\rightarrow QY \\ QY &\rightarrow QC \end{aligned}$$

A derivation of word $aaaabbbbcccc$:

$$\begin{aligned} &\Longrightarrow aaaaabbbbCCCXQ \\ &\Longrightarrow aaaaabbbbCCCXY \\ &\Longrightarrow aaaaabbbbCCCQY \\ &\Longrightarrow aaaaabbbbCCCQC \\ &\Longrightarrow aaaaabbbbCCXQC \\ &\Longrightarrow aaaaabbbbCCXYC \\ &\Longrightarrow aaaaabbbbCCQYC \\ &\Longrightarrow aaaaabbbbCCQCC \end{aligned} \qquad\qquad \begin{aligned} &\Longrightarrow aaaaabbbCXYCQ \\ &\Longrightarrow aaaaabbbCQYCQ \\ &\Longrightarrow aaaaabbbCQCCQ \\ &\Longrightarrow aaaaabbbXQCCQ \\ &\Longrightarrow aaaaabbbXYCCQ \\ &\Longrightarrow aaaaabbbQYCCQ \\ &\Longrightarrow aaaaabbbQCCCQ \\ &\Longrightarrow aaaaabbbbCCCCQ \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abC \\ bQC &\rightarrow bbCC \\ C &\rightarrow c \end{aligned} \qquad\qquad \begin{aligned} CQ &\rightarrow XQ \\ XQ &\rightarrow XY \\ XY &\rightarrow QY \\ QY &\rightarrow QC \end{aligned}$$

A derivation of word $aaaabbbbcccc$:

$$\begin{aligned} &\Rightarrow aaaaabbbbCCCXQ \\ &\Rightarrow aaaaabbbbCCCXY \\ &\Rightarrow aaaaabbbbCCCQY \\ &\Rightarrow aaaaabbbbCCCQC \\ &\Rightarrow aaaaabbbbCCXQC \\ &\Rightarrow aaaaabbbbCCXYC \\ &\Rightarrow aaaaabbbbCCQYC \\ &\Rightarrow aaaaabbbbCCQCC \end{aligned}$$

$$\Rightarrow aaaaabbbbCXQCC$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{aligned}
S &\to aSQ & \qquad CQ &\to XQ \\
S &\to abC & \qquad XQ &\to XY \\
bQC &\to bbCC & \qquad XY &\to QY \\
C &\to c & \qquad QY &\to QC
\end{aligned}
$$

A derivation of word $aaaabbbbcccc$:

$$
\begin{aligned}
&\Rightarrow aaaaabbbbCCCXQ \\
&\Rightarrow aaaaabbbbCCCXY \\
&\Rightarrow aaaaabbbbCCCQY \\
&\Rightarrow aaaaabbbbCCCQC \\
&\Rightarrow aaaaabbbbCCXQC \\
&\Rightarrow aaaaabbbbCCXYC \\
&\Rightarrow aaaaabbbbCCQYC \\
&\Rightarrow aaaaabbbbCCQCC
\end{aligned}
$$

$$
\begin{aligned}
&\Rightarrow aaaaabbbbCXQCC \\
&\Rightarrow aaaaabbbbCXYCC
\end{aligned}
$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abC \\ bQC &\rightarrow bbCC \\ C &\rightarrow c \end{aligned} \qquad\qquad \begin{aligned} CQ &\rightarrow XQ \\ XQ &\rightarrow XY \\ XY &\rightarrow QY \\ QY &\rightarrow QC \end{aligned}$$

A derivation of word $aaaabbbbcccc$:

$$\begin{aligned} &\Longrightarrow aaaaabbbbCCCXQ \\ &\Longrightarrow aaaaabbbbCCCXY \\ &\Longrightarrow aaaaabbbbCCCQY \\ &\Longrightarrow aaaaabbbbCCCQC \\ &\Longrightarrow aaaaabbbbCCXQC \\ &\Longrightarrow aaaaabbbbCCXYC \\ &\Longrightarrow aaaaabbbbCCQYC \\ &\Longrightarrow aaaaabbbbCCQCC \end{aligned} \qquad \begin{aligned} &\Longrightarrow aaaaabbbbCXQCC \\ &\Longrightarrow aaaaabbbbCXYCC \\ &\Longrightarrow aaaaabbbbCQYCC \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abC \\ bQC &\rightarrow bbCC \\ C &\rightarrow c \end{aligned} \qquad \begin{aligned} CQ &\rightarrow XQ \\ XQ &\rightarrow XY \\ XY &\rightarrow QY \\ QY &\rightarrow QC \end{aligned}$$

A derivation of word $aaaabbbbcccc$:

$$\begin{aligned} &\Longrightarrow aaaaabbbbCCCXQ \\ &\Longrightarrow aaaaabbbbCCCXY \\ &\Longrightarrow aaaaabbbbCCCQY \\ &\Longrightarrow aaaaabbbbCCCQC \\ &\Longrightarrow aaaaabbbbCCXQC \\ &\Longrightarrow aaaaabbbbCCXYC \\ &\Longrightarrow aaaaabbbbCCQYC \\ &\Longrightarrow aaaaabbbbCCQCC \end{aligned} \qquad \begin{aligned} &\Longrightarrow aaaaabbbbCXQCC \\ &\Longrightarrow aaaaabbbbCXYCC \\ &\Longrightarrow aaaaabbbbCQYCC \\ &\Longrightarrow aaaaabbbbCQCCC \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abC \\ bQC &\rightarrow bbCC \\ C &\rightarrow c \end{aligned} \qquad \begin{aligned} CQ &\rightarrow XQ \\ XQ &\rightarrow XY \\ XY &\rightarrow QY \\ QY &\rightarrow QC \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\begin{aligned} &\Rightarrow aaaaabbbbCCCXQ \\ &\Rightarrow aaaaabbbbCCCXY \\ &\Rightarrow aaaaabbbbCCCQY \\ &\Rightarrow aaaaabbbbCCCQC \\ &\Rightarrow aaaaabbbbCCXQC \\ &\Rightarrow aaaaabbbbCCXYC \\ &\Rightarrow aaaaabbbbCCQYC \\ &\Rightarrow aaaaabbbbCCQCC \end{aligned} \qquad \begin{aligned} &\Rightarrow aaaaabbbbCXQCC \\ &\Rightarrow aaaaabbbbCXYCC \\ &\Rightarrow aaaaabbbbCQYCC \\ &\Rightarrow aaaaabbbbCQCCC \\ &\Rightarrow aaaaabbbbXQCCC \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abC \\ bQC &\rightarrow bbCC \\ C &\rightarrow c \end{aligned} \qquad \begin{aligned} CQ &\rightarrow XQ \\ XQ &\rightarrow XY \\ XY &\rightarrow QY \\ QY &\rightarrow QC \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\begin{aligned} &\Rightarrow aaaaabbbbCCCXQ \\ &\Rightarrow aaaaabbbbCCCXY \\ &\Rightarrow aaaaabbbbCCCQY \\ &\Rightarrow aaaaabbbbCCCQC \\ &\Rightarrow aaaaabbbbCCXQC \\ &\Rightarrow aaaaabbbbCCXYC \\ &\Rightarrow aaaaabbbbCCQYC \\ &\Rightarrow aaaaabbbbCCQCC \end{aligned} \qquad \begin{aligned} &\Rightarrow aaaaabbbbCXQCC \\ &\Rightarrow aaaaabbbbCXYCC \\ &\Rightarrow aaaaabbbbCQYCC \\ &\Rightarrow aaaaabbbbCQCCC \\ &\Rightarrow aaaaabbbbXQCCC \\ &\Rightarrow aaaaabbbbXYCCC \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abC \\ bQC &\rightarrow bbCC \\ C &\rightarrow c \end{aligned} \qquad \begin{aligned} CQ &\rightarrow XQ \\ XQ &\rightarrow XY \\ XY &\rightarrow QY \\ QY &\rightarrow QC \end{aligned}$$

A derivation of word $aaaabbbbcccc$:

$$\begin{aligned} &\Longrightarrow aaaaabbbbCCCXQ \\ &\Longrightarrow aaaaabbbbCCCXY \\ &\Longrightarrow aaaaabbbbCCCQY \\ &\Longrightarrow aaaaabbbbCCCQC \\ &\Longrightarrow aaaaabbbbCCXQC \\ &\Longrightarrow aaaaabbbbCCXYC \\ &\Longrightarrow aaaaabbbbCCQYC \\ &\Longrightarrow aaaaabbbbCCQCC \end{aligned} \qquad \begin{aligned} &\Longrightarrow aaaaabbbbCXQCC \\ &\Longrightarrow aaaaabbbbCXYCC \\ &\Longrightarrow aaaaabbbbCQYCC \\ &\Longrightarrow aaaaabbbbCQCCC \\ &\Longrightarrow aaaaabbbbXQCCC \\ &\Longrightarrow aaaaabbbbXYCCC \\ &\Longrightarrow aaaaabbbbQYCCC \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{aligned}
S &\rightarrow aSQ & \qquad CQ &\rightarrow XQ \\
S &\rightarrow abC & XQ &\rightarrow XY \\
bQC &\rightarrow bbCC & XY &\rightarrow QY \\
C &\rightarrow c & QY &\rightarrow QC
\end{aligned}
$$

A derivation of word $aaaaabbbbbccccc$:

$$
\begin{aligned}
&\Longrightarrow aaaaabbbbCCCXQ & \qquad &\Longrightarrow aaaaabbbbCXQCC \\
&\Longrightarrow aaaaabbbbCCCXY & &\Longrightarrow aaaaabbbbCXYCC \\
&\Longrightarrow aaaaabbbbCCCQY & &\Longrightarrow aaaaabbbbCQYCC \\
&\Longrightarrow aaaaabbbbCCCQC & &\Longrightarrow aaaaabbbbCQCCC \\
&\Longrightarrow aaaaabbbbCCXQC & &\Longrightarrow aaaaabbbbXQCCC \\
&\Longrightarrow aaaaabbbbCCXYC & &\Longrightarrow aaaaabbbbXYCCC \\
&\Longrightarrow aaaaabbbbCCQYC & &\Longrightarrow aaaaabbbbQYCCC \\
&\Longrightarrow aaaaabbbbCCQCC & &\Longrightarrow aaaaabbbbQCCCC
\end{aligned}
$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\to aSQ \\ S &\to abC \\ bQC &\to bbCC \\ C &\to c \end{aligned} \qquad\qquad \begin{aligned} CQ &\to XQ \\ XQ &\to XY \\ XY &\to QY \\ QY &\to QC \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\Rightarrow\ aaaaabbbbbCCCCC$$

$$\begin{aligned} &\Rightarrow\ aaaaabbbbCXQCC \\ &\Rightarrow\ aaaaabbbbCXYCC \\ &\Rightarrow\ aaaaabbbbCQYCC \\ &\Rightarrow\ aaaaabbbbCQCCC \\ &\Rightarrow\ aaaaabbbbXQCCC \\ &\Rightarrow\ aaaaabbbbXYCCC \\ &\Rightarrow\ aaaaabbbbQYCCC \\ &\Rightarrow\ aaaaabbbbQCCCC \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{array}{rcl}
S & \rightarrow & aSQ \\
S & \rightarrow & abC \\
bQC & \rightarrow & bbCC \\
C & \rightarrow & c
\end{array}
\qquad\qquad
\begin{array}{rcl}
CQ & \rightarrow & XQ \\
XQ & \rightarrow & XY \\
XY & \rightarrow & QY \\
QY & \rightarrow & QC
\end{array}
$$

A derivation of word $aaaaabbbbbccccc$:

$$
\begin{array}{l}
\Rightarrow \ aaaaabbbbbCCCCC \\
\Rightarrow \ aaaaabbbbbcCCCC
\end{array}
\qquad\qquad
\begin{array}{l}
\Rightarrow \ aaaaabbbbCXQCC \\
\Rightarrow \ aaaaabbbbCXYCC \\
\Rightarrow \ aaaaabbbbCQYCC \\
\Rightarrow \ aaaaabbbbCQCCC \\
\Rightarrow \ aaaaabbbbXQCCC \\
\Rightarrow \ aaaaabbbbXYCCC \\
\Rightarrow \ aaaaabbbbQYCCC \\
\Rightarrow \ aaaaabbbbQCCCC
\end{array}
$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{aligned}
S &\to aSQ & \qquad CQ &\to XQ \\
S &\to abC & \qquad XQ &\to XY \\
bQC &\to bbCC & \qquad XY &\to QY \\
C &\to c & \qquad QY &\to QC
\end{aligned}
$$

A derivation of word $aaaaabbbbbccccc$:

$$
\begin{aligned}
&\Rightarrow aaaaabbbbbCCCCC \\
&\Rightarrow aaaaabbbbbcCCCC \\
&\Rightarrow aaaaabbbbbccCCC
\end{aligned}
$$

$$
\begin{aligned}
&\Rightarrow aaaaabbbbCXQCC \\
&\Rightarrow aaaaabbbbCXYCC \\
&\Rightarrow aaaaabbbbCQYCC \\
&\Rightarrow aaaaabbbbCQCCC \\
&\Rightarrow aaaaabbbbXQCCC \\
&\Rightarrow aaaaabbbbXYCCC \\
&\Rightarrow aaaaabbbbQYCCC \\
&\Rightarrow aaaaabbbbQCCCC
\end{aligned}
$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\rightarrow aSQ \\ S &\rightarrow abC \\ bQC &\rightarrow bbCC \\ C &\rightarrow c \end{aligned} \qquad\qquad \begin{aligned} CQ &\rightarrow XQ \\ XQ &\rightarrow XY \\ XY &\rightarrow QY \\ QY &\rightarrow QC \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\begin{aligned} &\Rightarrow aaaaabbbbCXQCC \\ &\Rightarrow aaaaabbbbCXYCC \\ &\Rightarrow aaaaabbbbCQYCC \\ &\Rightarrow aaaaabbbbCQCCC \\ &\Rightarrow aaaaabbbbXQCCC \\ &\Rightarrow aaaaabbbbXYCCC \\ &\Rightarrow aaaaabbbbQYCCC \\ &\Rightarrow aaaaabbbbQCCCC \end{aligned}$$

$$\begin{aligned} &\Rightarrow aaaaabbbbbCCCCC \\ &\Rightarrow aaaaabbbbbcCCCC \\ &\Rightarrow aaaaabbbbbccCCC \\ &\Rightarrow aaaaabbbbbcccCC \end{aligned}$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$
\begin{aligned}
S &\rightarrow aSQ & \qquad CQ &\rightarrow XQ \\
S &\rightarrow abC & \qquad XQ &\rightarrow XY \\
bQC &\rightarrow bbCC & \qquad XY &\rightarrow QY \\
C &\rightarrow c & \qquad QY &\rightarrow QC
\end{aligned}
$$

A derivation of word $aaaaabbbbbccccc$:

$$
\begin{aligned}
&\Rightarrow aaaaabbbbCXQCC \\
&\Rightarrow aaaaabbbbCXYCC \\
&\Rightarrow aaaaabbbbCQYCC \\
&\Rightarrow aaaaabbbbCQCCC \\
&\Rightarrow aaaaabbbbXQCCC \\
&\Rightarrow aaaaabbbbXYCCC \\
&\Rightarrow aaaaabbbbQYCCC \\
&\Rightarrow aaaaabbbbQCCCC \\
&\Rightarrow aaaaabbbbbCCCCC \\
&\Rightarrow aaaaabbbbbcCCCC \\
&\Rightarrow aaaaabbbbbccCCC \\
&\Rightarrow aaaaabbbbbcccCC \\
&\Rightarrow aaaaabbbbbccccC
\end{aligned}
$$

Context-sensitive Grammars

A context-sensitive grammar generating language $L = \{a^n b^n c^n \mid n \geq 1\}$

$$\begin{aligned} S &\to aSQ \\ S &\to abC \\ bQC &\to bbCC \\ C &\to c \end{aligned} \qquad\qquad \begin{aligned} CQ &\to XQ \\ XQ &\to XY \\ XY &\to QY \\ QY &\to QC \end{aligned}$$

A derivation of word $aaaaabbbbbccccc$:

$$\begin{aligned} &\Longrightarrow aaaaabbbbCXQCC \\ &\Longrightarrow aaaaabbbbCXYCC \\ &\Longrightarrow aaaaabbbbCQYCC \\ &\Longrightarrow aaaaabbbbCQCCC \\ &\Longrightarrow aaaaabbbbXQCCC \\ &\Longrightarrow aaaaabbbbXYCCC \\ &\Longrightarrow aaaaabbbbQYCCC \\ &\Longrightarrow aaaaabbbbQCCCC \end{aligned}$$

$$\begin{aligned} &\Longrightarrow aaaaabbbbbCCCCC \\ &\Longrightarrow aaaaabbbbbcCCCC \\ &\Longrightarrow aaaaabbbbbccCCC \\ &\Longrightarrow aaaaabbbbbcccCC \\ &\Longrightarrow aaaaabbbbbccccC \\ &\Longrightarrow aaaaabbbbbccccc \end{aligned}$$

Context-free Grammars

Another special type of generative grammars are **context-free grammars**.

Context-free grammars are also called **type-2** grammars.

A grammar $\mathcal{G} = (\Pi, \Sigma, S, P)$ is **context-free** if all its rules are of the form

$$X \rightarrow \gamma$$

where $X \in \Pi$, $\gamma \in (\Pi \cup \Sigma)^*$.

A example of a rule:

$$C \rightarrow DaBBc$$

Context-free Grammars

Remark: Not every context-free grammar is context-sensitive since a context-free grammar can contain also some other ε-rules (i.e., rules of the form $X \rightarrow \varepsilon$) in addition to $S \rightarrow \varepsilon$.

Arbitrary context-free grammar without ε-rules (resp. with at most one ε-rule $S \rightarrow \varepsilon$ where nonterminal S does not occur on the right-hand side of any rule) is a special case of a context-sensitive grammar.

For every context-free grammar $\mathcal{G}$, it is possible an equivalent context-free grammar without ε-rules.

So for every context-free grammar, there is an equivalent context-sensitive grammar.

Regular Grammars

Let us recall that a grammar is a **right** (resp. **left**) **regular** grammar if all its rules are of the following forms:

- $A \to wB$ (resp. $A \to Bw$)
- $A \to w$

where $A, B \in \Pi$, $w \in \Sigma^*$.

A grammar is **regular** if it is a right or left regular grammar.

Regular grammar are denoted as **type 3** grammars.

It is obvious that regular grammars are a special case of context-free grammars.

Chomsky Hierarchy

So according to the types of rules that can be used in a grammar, the grammars can be divided into these four types:

- **Type-0** — General **generative grammars**

  no restrictions on the rules

- **Type-1** — **Context-sensitive grammars**

  rules of the form $\alpha X\beta \to \alpha\gamma\beta$, where $|\gamma| \geq 1$
  (An exception is possible rule $S \to \varepsilon$, but then S does not occur on the right-hand side of any rule.)

- **Type-2** — **context-free grammars**

  rules of the form $X \to \gamma$

- **Type-3** — **regular grammars**

  rules of the form $X \to wY$ (resp. $X \to Yw$) or $X \to w$

where $\alpha, \beta, \gamma \in (\Pi \cup \Sigma)^*$, $X \in \Pi$, and $w \in \Sigma^*$

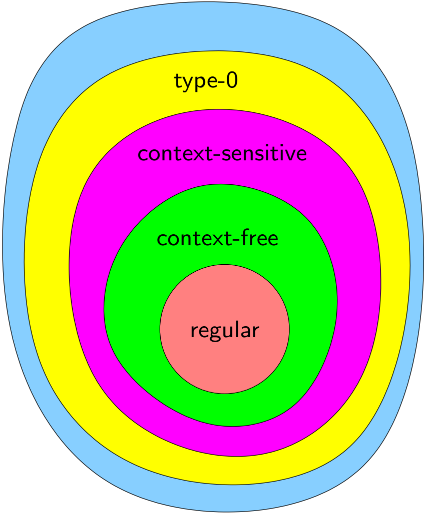
Chomsky Hierarchy

For all these types of grammars, there are corresponding classes of languages:

- **Type-0**: Language L is **recursively enumerable** (or of **type-0**) if there exists a generative grammar generating this language.
- **Type-1**: Language L is **context-sensitive** (or of **type-1**) if there exists a context-sensitive grammar generating this language.
- **Type-2**: Language L is **context-free** (or of **type-2**) if there exists a context-free grammar generating this language.
- **Type-3**: Language L is **regular** (or of **type-3**) if there exists a regular grammar generating this language.

Chomsky Hierarchy

Classes of languages:



Chomsky Hierarchy

- An example of a language that is context-free but is not regular:

$$\{a^n b^n \mid n \geq 1\}$$

- An example of a language that is context-sensitive but is not context-free:

$$\{a^n b^n c^n \mid n \geq 1\}$$

Chomsky Hierarchy

- Examples of languages that are type-0 but are not context-sensitive:
  - A language consisting of words that represent logically valid formulas of predicate logic.
  - Language consisting of words that represent codes of those Turing machines that will halt in a computation over an empty word after a finite number of steps.
- Examples of languages that are not of type-0:
  - A language consisting of those words that represent exactly those formulas of predicate logic, which are not logically valid.
  - A language constisting of words that represent codes of those Turing machines that never halt in a computation over an empty word.
  - A language consisting of words that represent codes of those Turing machines that will always halt after some finite number of steps in a computation over an arbitrary word.

Chomsky Hierarchy

- Other possible characterizations of **regular** languages:
  - languages accepted by finite automata (deterministic, nondeterministic, generalized nondeterministic)
  - languages that can be described by regular expressions
- Other possible characterization of **context-free** languages:
  - languages accepted by nondeterministic pushdown automata
- Other possible characterization of **context-sensitive** languages:
  - languages accepted by nondeterministic linear bounded automata
- Other possible characterization of **type-0** languages:
  - languages accepted by (deterministic or nondeterministic) Turing machines

Chomsky Hierarchy

Chomsky hierarchy — summary:

- **Type-0** — **recursively enumerable** languages:
  - unrestricted generative grammars
  - Turing machines (deterministic, nondeterministic)
- **Type-1** — **context-sensitive** languages:
  - context-sensitive grammars
  - nondeterministic linear bounded automata
- **Type-2** — **context-free** languages:
  - context-free grammars
  - nondeterministic pushdown automata
- **Type-3** — **regular** languages:
  - regular grammars
  - finite automata (deterministic, nondeterministic)
  - regular expressions