Exercise 4 – solution of the hyperintensional variants

1. **Analyse** the following argument (a) intensionally, (b) hyperintensionally and **explain** under which analysis it comes out valid**:**

Tom seeks an abominable snowman.
Abominable snowman is Yeti.
_____
Tom seeks Yeti.

Comment. The second premise is to be read *de dicto*, i.e. as specifying the *identity of the property of being an abominable snowman* and *being a Yeti*, but the terms 'abominable snowman' and 'Yeti' are not synonymous.

ad b) *Seek* is here a relation-in-intension of an individual to the *construction* of a property, the instances of which Tom wants to find; hence *Seek**/$(o\iota{*_n})_{\tau\omega}$.

Solution.

$\lambda w \lambda t$ [*Seek**$_{wt}$ ^{0}*Tom* ^{0}[^{0}*Abominable* ^{0}*Snowman*]]
$\lambda w \lambda t$ [^{0}= ^{0}*Yeti* [^{0}*Abominable* ^{0}*Snowman*]]

$\lambda w \lambda t$ [*Seek**$_{wt}$ ^{0}*Tom* ^{00}*Yeti*]

*Types. Seek**/$(o\iota{*_n})_{\tau\omega}$; *Tom*/$\iota$; *Abominable*/$((o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$: property modifier; *Snowman, Yeti*/$(o\iota)_{\tau\omega}$;

Type-checking. ^{0}[^{0}*Abominable* ^{0}*Snowman*] $\rightarrow$ $*_n$; [^{0}*Abominable* ^{0}*Snowman*] $\rightarrow$ $(o\iota)_{\tau\omega}$; =/$(o(o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$; [*Seek**$_{wt}$ ^{0}*Tom* ^{0}[^{0}*Abominable* ^{0}*Snowman*] $\rightarrow$ o; [^{0}= ^{0}*Yeti* [^{0}*Abominable* ^{0}*Snowman*]] $\rightarrow$ o.

The argument is *invalid* because the second premise establishes only the identity of properties but not synonymy. In order for the argument to be valid, the second premise would have to be [^{0}=* ^{00}*Yeti* ^{0}[^{0}*Abominable* ^{0}*Snowman*]], where =* is the relation of procedural isomorphism between constructions. But ^{0}*Yeti* and [^{0}*Abominable* ^{0}*Snowman*] are not procedurally isomorphic constructions, though they are equivalent.

2. **Prove the validity of the following argument** for both readings, intensional as well as hyperintensional**:**

Tom seeks an abominable snowman.
_____
Tom seeks something abominable.

Hint. In the intensional case, we do not have to apply the substitution method, but in the hyperintensional case we have to.

Solution.

$\lambda w \lambda t$ [*Seek**$_{wt}$ ^{0}*Tom* ^{0}[^{0}*Abominable* ^{0}*Snowman*]]

$\lambda w \lambda t$ [$^0\exists \lambda c$ [*Seek**$_{wt}$ ^{0}*Tom c*]]

Additional type. $c \rightarrow *_n$.

Proof.

1.	$[Seek*_{wt}\ {}^0Tom\ {}^0[{}^0Abominable\ {}^0Snowman]]$	$\varnothing$
2.	$\lambda c\ [[Seek*_{wt}\ {}^0Tom\ c]\ {}^0[{}^0Abominable\ {}^0Snowman]]$	λ-expansion, 1.
3.	$[{}^0\exists\lambda c\ [Seek*_{wt}\ {}^0Tom\ c]]$	$\exists$I, 2.

Comment. The step (3) is justified, as the class produced by $\lambda c\ [Seek*_{wt}\ {}^0Tom\ c]$ is non-empty; according to (2), it contains at least the construction $[{}^0Abominable\ {}^0Snowman]$.

3. Analyse and prove the validity:

Tom is solving the equation *Sin(x)=0*.

Tom is solving something.

Hint. Apply $Solve*/(o\iota*_n)_{\tau\omega}$; the relation-in-intension of an individual (here Tom) to the *construction*. Tom wants to find out what is constructed by that construction. The term 'equation *Sin(x)=0*' can be analysed simply by the Closure $\lambda x\ [[{}^0Sin\ x] = {}^00]$. To infer the conclusion, you do not have to apply the substitution method.

Solution is quite similar to the solution ad (2).

$\lambda w\lambda t\ [Solve*_{wt}\ {}^0Tom\ {}^0[\lambda x\ [[{}^0Sin\ x] = {}^00]]]$

$\lambda w\lambda t\ [{}^0\exists\lambda c\ [Solve*_{wt}\ {}^0Tom\ c]]$

Additional type. $c \rightarrow *_n$.

The *proof* is quite similar to the proof of (2). The witness of non-emptiness of the set produced by $\lambda c\ [Solve*_{wt}\ {}^0Tom\ c]$ is the construction $[\lambda x\ [[{}^0Sin\ x] = {}^00]]$.

4. Analyse and prove the validity:

Tom is solving the equation *Sin(x)=0*.

There is a number *n* such that Tom is solving the equation *Sin(x)=n*.

Hint. To infer the conclusion, you *must* apply the substitution method.

Solution.

$\lambda w\lambda t\ [Solve*_{wt}\ {}^0Tom\ {}^0[\lambda x\ [[{}^0Sin\ x] = {}^00]]]$

$\lambda w\lambda t\ [{}^0\exists\lambda n\ [Solve*_{wt}\ {}^0Tom\ [{}^0Sub\ [{}^0Tr\ n]\ {}^0m\ {}^0[\lambda x\ [[{}^0Sin\ x] = m]]]]]$

Types. $m,n \rightarrow \tau$; $Sub/(*_n*_n*_n*_n)$; $Tr/(*_n\tau)$.

Remark. The conclusion must not be of the form $\lambda w\lambda t\ [{}^0\exists\lambda n\ [Solve*_{wt}\ {}^0Tom\ {}^0[\lambda x\ [[{}^0Sin\ x] = n]]]]$ because the variable *n* occurs within the hyperintensional context; it is bound by Trivialization. Hence, its valuation is irrelevant. In other words, Tom isn't solving the Closure $^0[\lambda x\ [[{}^0Sin\ x] = n]]$.

Proof.

1.	$[Solve*_{wt}\ {}^0Tom\ {}^0[\lambda x\ [[{}^0Sin\ x] = {}^00]]]$	$\varnothing$
2.	$[\lambda n\ [Solve*_{wt}\ {}^0Tom\ [{}^0Sub\ [{}^0Tr\ n]\ {}^0m\ {}^0[\lambda x\ [[{}^0Sin\ x] = m]]]\ {}^00]$	λ-expansion, 1
3.	$[{}^0\exists\lambda n\ [Solve*_{wt}\ {}^0Tom\ [{}^0Sub\ [{}^0Tr\ n]\ {}^0m\ {}^0[\lambda x\ [[{}^0Sin\ x] = m]]]]$	

Comment. The step (2) is justified because the β-reduction of (2) produces

$$[Solve*_{wt}\ {}^0Tom\ [{}^0Sub\ [{}^0Tr\ {}^00]\ {}^0m\ {}^0[\lambda x\ [[{}^0Sin\ x] = m]]],$$

which is equivalent to

$$[Solve*_{wt}\ {}^0Tom\ {}^0[\lambda x\ [[{}^0Sin\ x] = {}^00]]].$$