

Tutorial 5 – Set Theory.

Exercise 1: Define, make an example and draw:

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|---|----------------------------|
| a) Union of sets A and B. | $(A \cup B)$ |
| b) Intersection of sets A and B. | $(A \cap B)$ |
| c) Difference of sets A and B. | $(A \setminus B)$ |
| d) A set A is a subset of a set B. | $(A \subseteq B)$ |
| e) A complement of a set A with respect to a set M. | $\overline{A}$ |
| f) Equality of sets A and B. | $(A = B)$ |
| g) A power set of a set A. | $P(A)$ nebo 2^A |
| h) A is a proper subset of B. | $(A \subset B)$ |
| i) Cartesian product of A and B | $(A \times B)$ |
| j) A relation between sets A and B. | $(R \subseteq A \times B)$ |
| k) Mapping from A to B. | $(A \rightarrow B)$ |

Exercise 2: Prove for any set A and B:

- $A = B$, iff $A \subseteq B$ a $B \subseteq A$
- $A \subseteq (A \cup B)$
- $(A \cap B) \subseteq A$
- If $A = \emptyset$ and $B = \emptyset$ then $(A \cup B) = \emptyset$ and vice versa.
- If $A = \emptyset$ then $(A \cap B) = \emptyset$ for any set B.
- $((A \cup B) \setminus C) \subseteq (A \cup (B \setminus C))$
- $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$ de Morgann's laws
- $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$ de Morgann's laws

Exercise 3: Decide, whether following statements are true:

- $a \in \{\{a\}, \{a, \{a\}\}\}$
- $\{a, \{a\}\} \cap P(\{a, \{a\}\}) = \emptyset$
- $\{\emptyset\} \in \{\{\emptyset\}\}$

d) $\bigcap A_i = \emptyset$, where A_i is an element of $\mathcal{P}(A)$, i.e. $A_i \in 2^A$.

Exercise 4: Identify all elements of following sets:

- a) $\{a, \{a\}\} \cup \{a, \{b\}, c\}$
- b) $\{a, \{a\}\} \cap \{a, \{b\}, c\}$
- c) $\{a, \{a\}\} \setminus \{a, \{b\}, c\}$
- d) $\overline{A}$ with respect to B where $A = \{a, b, \{c\}\}$, $B = \{a, b, \{c\}, \{a, b\}\}$ and $A \subseteq B$

Exercise 5: Decide, whether following statements are true:

- a) A set of people living in Europe or Asia and not in Ural mountains is a subset of people living in Europe or Asia.
- b) All citizens of Czech republic live in Moravia, Bohemia or in Silesia.
- c) Some citizens of Moravia live in Silesia.
- d) All people living in Europa and Asia belong into a set of people living in Asia or Europe.

Exercise 6: Decide, whether the following relations are functions, if so which kind. We will work with sets: $A = \{a_1, a_2, a_3, a_4\}$, $B = \{b_1, b_2, b_3\}$ a $C = \{c_1, c_2, c_3\}$:

- a) $R \subseteq A \times B$, $R = \{(a_1, b_3), (a_2, b_2), (a_1, b_1)\}$
- b) $R \subseteq B \times A$, $R = \{(b_1, a_4), (< b_2, a_4), (b_3, a_4)\}$
- c) $R \subseteq A \times C$, $R = \{(a_1, c_2), (< a_2, c_3), (a_3, c_1)\}$
- d) $R \subseteq C \times A$, $R = \{(c_2, a_1), (c_3, a_2), (c_1, a_3)\}$
- e) $R \subseteq B \times C$, $R = \{(b_1, c_3), (b_2, c_2), (b_3, c_1)\}$
- f) $R \subseteq A \times B \times C$, $R = \{(a_1, b_1, c_1), (a_2, b_2, c_1), (a_1, b_2, c_3)\}$
- g) $R = \{< x, y > \in \mathbb{N} \times \mathbb{N}, y = x^2\}$
- h) $R = \{< x, y > \in \mathbb{N} \times \mathbb{N}, x = y^2\}$
- i) $R = \{< x, y > \in \mathbb{Z} \times \mathbb{Z}, y = x^2\}$
- j) $R = \{< x, y > \in \mathbb{Z} \times \mathbb{Z}, x = y^2\}$