VSB TECHNICAL UNIVERSITY OF OSTRAVA | FACULTY OF ELECTRICAL ENGINEERING AND COMPUTER SCIENCE | DEPARTMENT OF COMPUTER SCIENCE

Dynamic Programming

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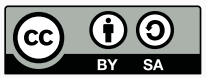

Lecture outline

Dynamic Programming

Warshall's algorithm

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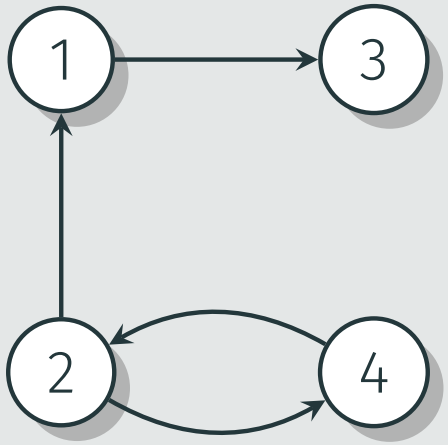
Warshall's algorithm

Warshall's algorithm

$$\mathbf{R}_{i,j}^{(k)} = \begin{cases} \mathbf{R}_{i,j}^{(k-1)} & \text{path through vertices } 1, \ldots, k-1 \\ \vee & \\ \mathbf{R}_{i,k}^{(k-1)} \wedge \mathbf{R}_{k,j}^{(k-1)} & \text{paths from } i \text{ to } k \text{ and from } k \text{ to } j \text{ through vertices } 1, \ldots, k-1 \end{cases}$$

Warshall's algorithm – example

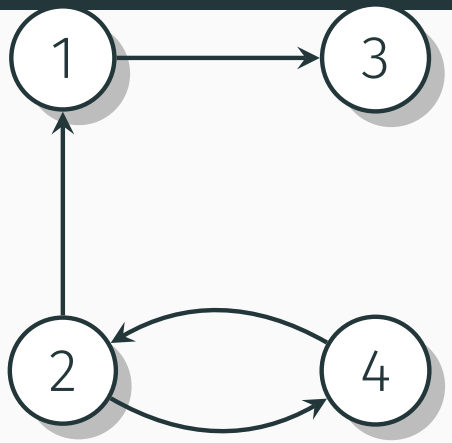
Sample graph G

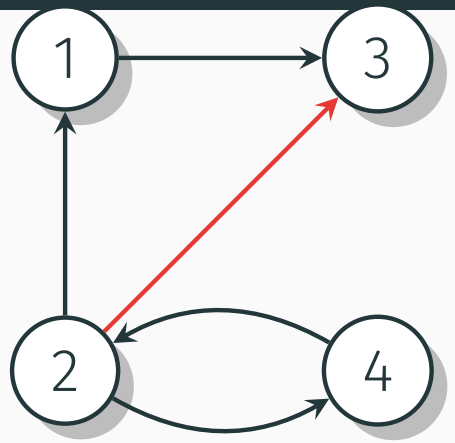


Adjacency matrix

$$\mathbf{A}_G = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Warshall's algorithm – example, $k = 1$



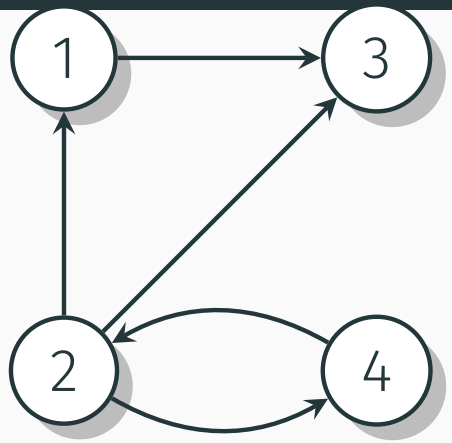


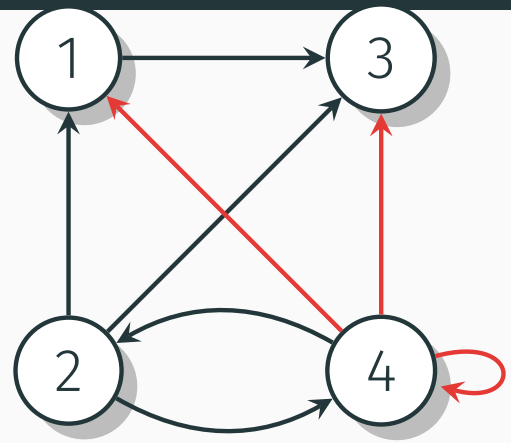
$$\mathbf{R}^{(0)} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\mathbf{R}^{(1)} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Edges for paths leading through vertex 1 have been added to the graph.

Warshall's algorithm – example, $k = 2$



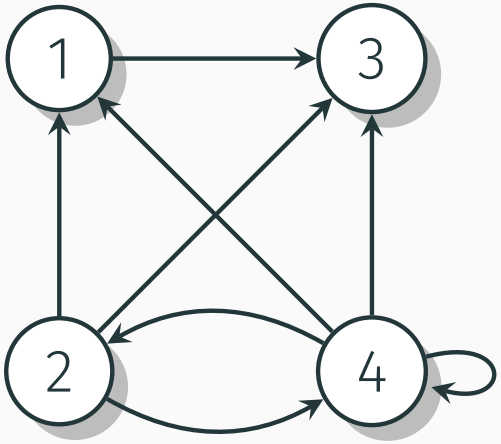


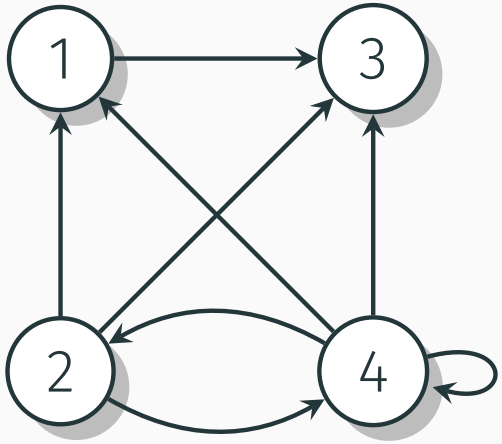
$$\mathbf{R}^{(1)} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\mathbf{R}^{(2)} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Edges for paths leading through vertex 2 have been added to the graph.

Warshall's algorithm – example, $k = 3$

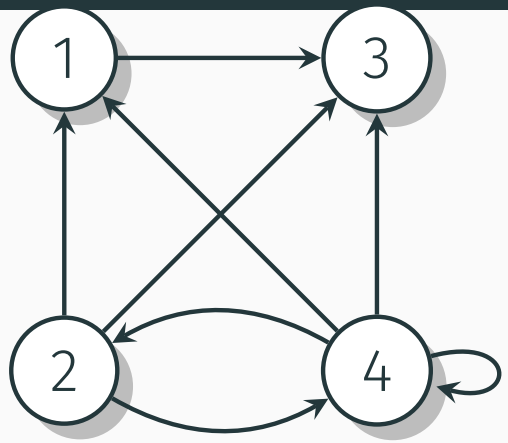


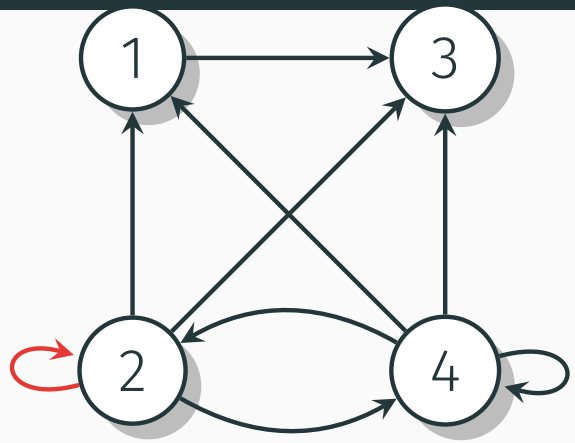


$$\mathbf{R}^{(2)} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} \qquad \mathbf{R}^{(3)} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

No edge was added to the graph – **through** vertex 3 no path leads, edges lead only **to** vertex 3.

Warshall's algorithm – example, $k = 4$





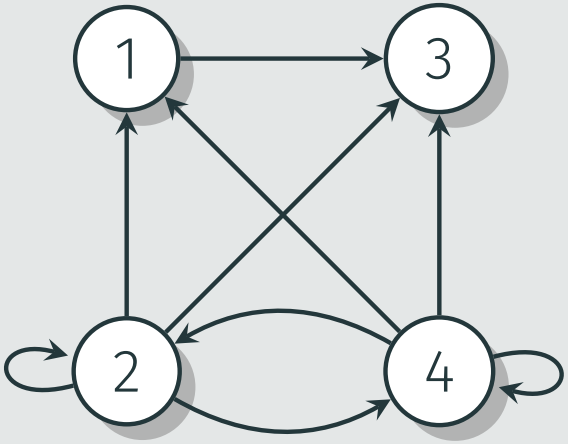
$$\mathbf{R}^{(3)} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\mathbf{R}^{(4)} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Edges for paths leading through vertex 4 have been added to the graph.

Warshall's algorithm – example

Resulting transitive closure T



Adjacency matrix

$$\mathbf{A}_T = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Thanks for your attention